1. Suppose that $a$ is a positive constant and that $R$ is the region bounded above by $y = 1/x^a$, below by $y = 0$, and on the left by the line $x = 1$.

a) Sketch the curves $y = 1/x^a$ for $a = .5, 1$ and $2$. Which of these is closest to the $x$-axis?

b) For which positive numbers $a$ do you get a convergent integral when you attempt to calculate the area of $R$?

c) Same as b), but for the volume of the solid obtained by rotating $R$ around the $x$-axis.

d) Same as c), but for the volume of the solid obtained by rotating $R$ around the $y$-axis.

2. Sketch carefully the graphs of $f(x) = (1 + e^{-x})^2$ and $g(x) = (1 + e^{-2x})^2$ for $x > 0$, and compute how much area there is between them in the first quadrant.

3. Sketch the three-sided region in the first quadrant bounded by the $y$-axis and the two curves $y = \tan x$ and $y = \sec x$. Compute the area of this region.

4. In class we considered the improper integral $\int_1^\infty \frac{\ln(x)}{x^2} dx$. We decided that this integral converges and its value is 1.

a) Make the change of variable $w = \ln x$ and remember also to change the bounds on the definite integral. What is the resulting definite integral and what is its value?

b) Make the change of variable $w = \frac{1}{x}$ in the original integral and remember also to change the bounds on the definite integral. What is the resulting definite integral and what is its value?

About these workshop problems The second and third problems are taken from past Calc 2 exams. The answer to the fourth problem’s part a) resembles a problem on the exam you just had.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield’s Math 152 webpage to learn which problem to hand in.