1. The function $S$ (the “squaring function”) has domain all real numbers and is defined by the formula $S(x) = x^2$ for all $x$.
   a) Consider the function $T$ whose domain is also all real numbers which is defined by
   \[ T(x) = \begin{cases} 
   S(x) & \text{if } x \neq 3, \\
   7 & \text{if } x = 3.
   \end{cases} \]
   Sketch a graph of $T$. What is $\lim_{x \to 5} T(x)$? What is $\lim_{x \to 3} T(x)$? Support your assertions.
   b) An evil interstellar visitor changes exactly one million values of $S$ and creates a new function, $V$. What can be said about $\lim_{x \to a} V(x)$ for all values of $a$? Support your assertions.

2. Use your calculator to try to figure out what might be the value of
   \[ \lim_{x \to 0} \frac{\cot x}{x} \left(1 - \cos 2x\right) \]
   by tracing the graph of $\frac{(\cot x)(1-\cos 2x)}{x}$ for $x$ near 0. Show the graph. Then find the exact value of the limit by using a computation based on trig formulas and the fact that $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$.

3. $f(x)$ is a piecewise function defined as follows: $f(x) = \begin{cases} 
   2x^2 + 2, & \text{if } x < 1, \\
   ax^2 + bx, & \text{if } 1 \leq x \leq 2, \\
   2 - \frac{6}{x}, & \text{if } x > 2.
   \end{cases}$
   a) Suppose that $a = 2$ and $b = -3$. Graph $f(x)$ for $0 \leq x \leq 3$. Find the left and right hand limits of $f(x)$ as $x$ approaches 1 and as $x$ approaches 2.
   b) Find $a$ and $b$ so that the graph of $f(x)$ doesn’t have any jumps (that is, $f(x)$ is continuous everywhere). Graph the resulting function $f(x)$ for $0 \leq x \leq 3$.

4. About a decade ago three centuries of effort by mathematicians culminated in a proof that there were no solutions to the Fermat equation $a^n + b^n = c^n$ if $a$, $b$, $c$, and $n$ are positive integers, with $n > 2$. There are, of course, solutions when $n = 2$: for example, $3^2 + 4^2 = 5^2$.
   a) Does the equation $4^x + 5^x = 6^x$ have any solution? (The word “integer” does not appear in the preceding sentence!)
   b) Suppose $a$, $b$, and $c$ are positive real numbers. Explore whether the equation
   \[ a^x + b^x = c^x \]
   must have a solution. This is a “free form” question: try to answer it as well as you can. You are not asked to provide a “formula” for $x$. You are asked to find conditions which will guarantee that such an $x$ either does or does not exist.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield’s Math 151 webpage to learn which problem to hand in.