1. In this problem, \( f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 \).
Find the absolute minimum and absolute maximum values of \( f \) in the interval \([-2, 1]\).

2. The program Maple displays the image shown to the right when asked to graph the equation
\[ y^2 = x^3 - 3xy + 3. \]
a) Verify by substitution that the point \( P = (-2, 1) \) is on the graph of the equation.
b) Find \( \frac{dy}{dx} \) in terms of \( y \) and \( x \).
c) Find an equation for the line tangent to the graph at the point \( P = (-2, 1) \). You do not need to “simplify” your answer!
d) Sketch this tangent line in the appropriate place on the image displayed.

3. Ant A is crawling up a vertical pole at .3 meters/minute. At the same time ant B is crawling away from the base of the pole on the horizontal ground at .4 meters/minute.
a) If \( \theta \) is the angle that ant B sees between the base of the pole and ant A, if \( a \) is the distance from ant A to the base of the pole, and if \( b \) is the distance from ant B to the base of the pole, then write a formula for \( \theta \) as a function of \( a \) and \( b \).
b) Use the information provided to compute \( \theta \) and \( \frac{d\theta}{dt} \) at the instant that ant A is 10 meters up the pole and ant B is 5 meters from the base of the pole. You do not need to “simplify” your answer!

4. Suppose \( B(x) \) is a differentiable function with \( B(2) = 3 \) and that the derivative of \( B \) is given by the following formula:
\[ B'(x) = \sqrt{23 - 7x}. \]
Suppose also that \( C(x) = 5x^2 - 3 \). Let \( A(x) = B(C(x)) \).
a) Compute \( A(1) \). Write a formula for \( A'(x) \) only in terms of \( x \) and then compute \( A'(1) \).
b) Use your answers to a) and linear approximation to find an approximate value of \( A(0.95) \). You do not need to “simplify” your answer!
c) It is true that \( A''(1) = -\frac{260}{3} \). Is the estimate you found in a) likely to be greater than or less than the true value of \( A(0.95) \)? Give reasoning which supports your answer.
5. In the right triangle $\triangle ABC$, the right angle is at $C$ and the legs are $|AC| = 4$ and $|BC| = 12$. A rectangle is placed inside the triangle, with one corner at $C$ and the opposite corner on the hypotenuse. What are the dimensions and area of the rectangle which has largest area? Briefly explain why you found the rectangle with largest area.

6. Find the limits. 
   a) $\lim_{x \to 1} \frac{x^4 - 4x + 3}{(x - 1)^2}$. 
   b) $\lim_{x \to \infty} (5 + 3x)^{2/x}$. 
   c) $\lim_{x \to \infty} \frac{\pi - \arctan x}{e^x - 1}$.

7. In this problem, $f(x) = \frac{x + 1}{x^2 + 3}$.
   a) What are $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$? $\lim_{x \to +\infty} f(x) = \boxed{\;}$ and $\lim_{x \to -\infty} f(x) = \boxed{\;}$.
   b) Compute $f'(x)$ carefully, since the result is needed for successful completion of the remainder of the problem. Simplify your result. $f'(x) = \boxed{\;}$.
   c) Find all solutions of $f'(x) = 0$. For each such $x$, compute $f(x)$.
      $x = \boxed{\;}$ and $f(\boxed{\;}) = \boxed{\;}$. $x = \boxed{\;}$ and $f(\boxed{\;}) = \boxed{\;}$.
   d) Where is $f'(x) > 0$? Where is $f'(x) < 0$? $f'(x) > 0$ when $\boxed{\;}$ and $f'(x) < 0$ when $\boxed{\;}$.
   e) Sketch a graph of $y = f(x)$. The conclusions of parts a) and c) and d) should all be used here. The scales of the vertical and horizontal axes are very different.

   \[ ...\]

   f1) How many solutions does the equation $f(x) = .07$ have? (You are not asked to find the solutions!)$f(x) = .07$ has $\boxed{\;}$ solutions.

   f2) How many solutions does the equation $f(x) = .87$ have? (You are not asked to find the solutions!)$f(x) = .87$ has $\boxed{\;}$ solutions.

   g) What is the range of $f$? (That is, the collection of all $y$'s for which $f(x) = y$ has at least one solution.) The range of $f$ is $\boxed{\;}$.

   This problem continues on the next page.
h) You do not need to compute $f''(x)$ to answer the following question: how many inflection points must $y = f(x)$ have, and what can you say about the approximate location of these inflection points?

The number of inflection points is __________. Show each of the approximate values of their first coordinates on the number line below with a ●.

\[
\begin{array}{cccccccc}
-10 & -5 & -1 & 0 & 1 & 5 & 10 \\
\end{array}
\]

(6) 8. Suppose that $f(x)$ is a differentiable function, and that for all $x$, $4 < f'(x) < 6$. Suppose also that $f(0) = -2$.

a) Explain why $f(5)$ must be positive. You should quote a specific result from this course and explain its relevance.

b) Explain why $f(x) = 0$ must have a solution in the interval $[0, 5]$. You may assume the sign information stated in a) here. You should quote a specific result from this course and explain its relevance.
Second Exam for Math 151
Section 4 or 5 or 6

November 21, 2006

NAME _____________________________________________

SECTION _______

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.

No texts, notes, or calculators other than the
formula sheet supplied may be used on this exam.

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Total Points Earned: ________________________________