1 B

(8) 1. A point is moving along the graph of the function \( y = \sin x \) so that \( \frac{dx}{dt} \) is 2 centimeters per second. Find \( y \) and \( \frac{dy}{dt} \) when \( x = \frac{\pi}{6} \).

\[
y = \quad \text{__________________________}
\]

\[
\frac{dy}{dt} = \quad \text{__________________________}
\]

(10) 2. Find the indicated limits. Give evidence to support your answers.

a) \( \lim_{x \to +\infty} \frac{3x + 2}{2x - 1} \)

b) \( \lim_{x \to \frac{1}{2}^+} \frac{3x + 2}{2x - 1} \)
(12) 3. Find all relative maximum and minimum values of the function \( f(x) = (x^2 - 3)e^x \). Briefly explain your answers using calculus.
(14) 4. The program Maple displays this image when asked to graph the equation

\[ y^2 = x^3 - 3xy + 3. \]

a) Verify by substitution that the point \( P = (-2, 1) \) is on the graph of the equation.

b) Find \( \frac{dy}{dx} \) in terms of \( y \) and \( x \).

c) Find an equation for the line tangent to the graph at the point \( P = (-2, 1) \).

d) Sketch this tangent line in the appropriate place on the image displayed.
5. A rectangle is bounded by the $x$-axis and the semicircle $y = \sqrt{25 - x^2}$ (see figure). What length and width should the rectangle have so that its area is a maximum?

Briefly explain using calculus why your answer gives a maximum.
6. Suppose \( W(x) = -\frac{1}{2}x^2 + 6x - 5 \ln x. \)

a) Compute \( W'(x) \) and \( W''(x) \). Where are these functions equal to 0?

b) What is \( \lim_{x \to 0^+} W(x) \)? **ANSWER**

c) Sketch a graph on the axes given of \( y = W(x) \) for \( x \) between 0 and 7.

- Label any relative maxima with an **M** on your graph.
- Label any relative minima with an **m** on your graph.
- Label any points of inflection with an **I** on your graph.

- In what interval(s) on this graph is \( W(x) \)
  - increasing? **ANSWER**
  - decreasing? **ANSWER**
  - concave up? **ANSWER**
  - concave down? **ANSWER**
(20) 7. To the right is a graph of $h'(x)$, the derivative of a function, $h$. Use this graph to answer the questions below.

a) Use information from the graph of $h'(x)$ to find the $x$ where the maximum value of $h$ in the interval $1 \leq x \leq 3$ will occur. Briefly explain using calculus why your answer is correct, including verification that the value of $h$ at the $x$ you select is larger than $h$’s value at any other number in the interval.

b) Suppose that $h(3) = 5$. Use information from the graph and the differential or tangent line approximation to find an approximate value of $h(3.04)$. Briefly explain using calculus and information from the graph why your approximation for $h(3.04)$ is greater than or less than the exact value of $h(3.04)$. 
Exam 2 for Math 135
Sections 8, 9, and 10
April 13, 1999

NAME (please print):______________________________________________

SIGNATURE:________________________________________________________

SECTION #:________________________________________________________

Do all problems, in any order.
Show all your work. Full credit may not be given for an answer alone.
You may use one sheet of notes and any standard calculator without
a QWERTY keypad on this exam. You may use no other materials.

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