Math 403, section 5

Entrance “exam”

Due at the beginning of class, Monday, January 22, 2001

1. (8) Compute \( \int_{1}^{2} \frac{dx}{1-2x} \) and simplify.

2. (6) If \( u(x, y) = e^{y^2x} \), what are \( \partial u / \partial x \) and \( \partial u / \partial y \)?

3. (6) Find a function \( v(x, y) \) such that \( \partial v / \partial x = x + y \) and \( \partial v / \partial y = x - y \).

4. (6) What shape (square, line, disk, circle, etc.) is described by the collection of ordered pairs \( \{(2 + \cos \theta, \sin \theta)\} \) when \( 0 \leq \theta \leq \pi \)?

5. (10) Find all values of \( x \) for which the series \( \sum_{n=0}^{\infty} (x - 2)^n \) converges.

6. (6) The arctangent function, \( \arctan x \), is an antiderivative of \( \frac{1}{1 + x^2} \). Based on this fact or otherwise, find the exact value of \( \int_{0}^{4} \frac{dy}{1 + 4y^2} \).

7. (8) Estimate the value of \( \int_{0}^{1} x^2 + \frac{\sin(x^2)}{300} \, dx \) two decimal places. Explain your reasoning.

8. (10) Compute the line integral of \( x^2 \, dx + xy \, dy \) along two paths: the straight line from \((0, 0)\) to \((1, 2)\), and the parabolic arc \( y = 2x^2 \) from \((0, 0)\) to \((1, 2)\).

Rules You may treat this as any other homework assignment. You may consult textbooks or acquaintances, but the written work you hand in must be your own. An answer alone will not receive full credit – you must show supporting computation or give some explanation or both. I will grade what you hand in as an exam. A passing grade will be at least 75% of the 60 points.