Please write solutions to two of these problems. Hand them in Monday, March 24. The written solutions should be accompanied by explanations using complete English sentences. Students should work alone. They may ask me questions.

1. Use the Squeeze Theorem to give a simple proof that if \( k \in \mathbb{N} \), then \( \lim_{n \to \infty} \left( \frac{n^k}{n!} \right) \) exists and is 0.

2. Suppose that \((x_n)_{n \in \mathbb{N}}\) is a sequence with \( x_n > 0 \) for all \( n \), and that

\[
\lim_{n \to \infty} (x_n)^{1/n} = L \text{ exists.}
\]

If \( L < 1 \) prove that

\[
\lim_{n \to \infty} x_n = 0
\]

**Hint** Perhaps try this first with some number for \( L \), such as \( L = .9 \). What happens if you take \( \varepsilon = .05 \)? How big and how small can \((x_n)^{1/n}\) be when \( n \) is large enough (depending on \( \varepsilon \), of course)? What then will be true about \( x_n \) itself? Once you’ve done this, try to generalize the reasoning to every \( L < 1 \).

3. Suppose \((b_n)\) is a sequence with the following property:

\[†\] For any \( n \in \mathbb{N} \) there is an integer \( m \) with \( m > n \) so that \( a_m \cdot a_n < 0 \).

a) Find a simple example of a sequence satisfying \(†\).

b) Suppose in addition that \((a_n)\) converges, and its limit is \( A \). What can you say about \( A \)? Prove your assertion.

4. Suppose \((b_n)\) is a sequence with the following property:

\[⋆\] For any \( n \in \mathbb{N} \) there is an integer \( m \) with \( m > n \) so that \( b_m > b_n \).

a) Find a simple example of a sequence satisfying \(⋆\).

b) Are there any examples of non-monotonic sequences satisfying \(⋆\)? (Of course you must read \(⋆\) very carefully.) If there are, give one example. If there are not, prove your assertion.

c) Suppose in addition that \((b_n)\) converges, and its limit is \( B \). What can you say about the relation of \( B \) to any of the individual members of the sequence as a result of \(⋆\)?

5. Let \((a_n)\) be an infinite sequence. Assume:

\[
\lim_{n \to \infty} n^4 \left( a_n - 4 - \frac{5}{n^2} \right) = 12.
\]

Prove:

\[
\lim_{n \to \infty} n^2 (a_n - 4) = 5.
\]