1. Suppose the sequence \((x_n)\) is defined by 
\[ x_n := \frac{n-1}{5n+7}. \]
Find \(x\) so that \((x_n)\) converges to \(x\). Prove your assertion using the definition of convergence.

2. Suppose that the sequence \((x_n)\) converges to \(x\) and the sequence \((y_n)\) converges to \(y\). Prove that the sequence \((z_n)\) defined by 
\[ z_n := x_n + y_n \]
converges to \(x + y\).

3. Suppose \(S\) is a nonempty subset of \(\mathbb{R}\) which is bounded above, and \(a \in \mathbb{R}\). Define a subset \(T\) of \(\mathbb{R}\) by 
\[ T := \{ x : \exists s \in S \text{ so that } x = a + s \}. \]
\((T\) is \(S\) “translated by \(a\).\) Prove that \(T\) is bounded above, and prove that \(\sup T = a + \sup S\).

4. Prove that \(2n^2 < 3^n\) for all \(n \in \mathbb{N}\).

Comment You may need to verify more than one example numerically.

5. Suppose that \(S\) is a nonempty subset of \(\mathbb{R}\) with the property that if \(a \in S\) then \(a^2 \in S\). Prove that if \(S\) is bounded above, then \(\sup S \leq 1\).

6. Suppose that \((x_n)\) is a convergent sequence and \((y_n)\) is such that for any \(\varepsilon > 0\) there exists \(M(\varepsilon) \in \mathbb{N}\) such that 
\[ |x_n - y_n| < \varepsilon \]
for all \(n \geq M(\varepsilon)\). Does it follow that \((y_n)\) is convergent? Prove your assertion.

Answers to Part 1 of Exam 1

1. The definition of supremum
Suppose \(S\) is a nonempty subset of \(\mathbb{R}\). \(x\) is a supremum of \(S\) if
   a) \(x\) is an upper bound of \(S\): for \(s \in S\), \(s \leq x\).
   b) If \(y\) is an upper bound of \(S\), then \(x \leq y\).

2. A criterion for an upper bound to be a supremum
Suppose \(x\) is an upper bound of a nonempty subset of \(\mathbb{R}\). \(x\) is a supremum of \(S\) if and only if for all \(\varepsilon > 0\), there is \(s \in S\) with \(x - \varepsilon < s \leq x\).

3. The Completeness Axiom
If \(S\) is a nonempty subset of \(\mathbb{R}\) which is bounded above, then \(S\) has a least upper bound.

4. The Archimedean Property
If \(x \in \mathbb{R}\) then there is \(n \in \mathbb{N}\) so that \(n > x\).

5. The definition of convergence of a sequence
A sequence \((x_n)\) converges to \(x \in \mathbb{R}\) if for every \(\varepsilon > 0\) there is \(K(\varepsilon) \in \mathbb{N}\) such that for \(n \in \mathbb{N}\) with \(n \geq K(\varepsilon), |x_n - x| < \varepsilon\).
Do all problems, in any order.
No notes or texts may be used on this exam.
The last page contains the answers to Part 1.

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