

**This is the last set of problems for the semester!**

Please write solutions to two of these problems. Hand them in Thursday, November 21. The written solutions should be accompanied by explanations using complete English sentences. Students may work in groups of at most three, **but** students may only hand in work done with people with whom they have not previously handed in work.

1. A four foot long stick is broken into three pieces. One piece is painted red, one is painted green, and one is painted blue. What is the chance that the blue piece is at least one foot long at the same time that the red piece is more than twice as long as the green piece?

2. **The bat and the slug**

a) The average value of a function  $f(x, y, z)$  over a region  $\mathcal{R}$  in  $\mathbb{R}^3$  is  $\iiint_{\mathcal{R}} f(x, y, z) dv$  divided by  $\iiint_{\mathcal{R}} 1 dv$ . If a hollowed-out hemisphere of inner radius  $A$  and outer radius  $B$  is defined to be the region consisting of those points in  $\mathbb{R}^3$  which have third coordinate  $\geq 0$  and have distance to the origin  $\geq A$  and  $\leq B$ , what is the average height ( $z$  value) of a hollowed-out hemisphere?

b) *The bat flies ...* A bat flies in and around a hemispherical cave, with water at the bottom, so it cannot land there. If the radius of the cave is  $R$ , what is the average height of the cave to the bat?

I have been urged by a legalistic & unpoetic colleague to add: "The bat flies totally at random throughout all of the space available to it." The colleague then remarked (while talking to me), "Of course, they've had you all semester, so they know what to expect." What could she have meant?

Hint:  $A = 0$  and  $B = R$ .

c) *The slug crawls ...* A *non-swimming* slug crawls about on the inner surface of the same cave as in b). Its motion is confined to that surface of the cave. What is the average height of the cave to the slug?

Please add (sigh!): "The slug crawls totally at random throughout all of the space available to it."

Hint:  $A \rightarrow R^-$  and  $B = R$ .

d) Which creature is higher (on average)?\*

*Thank you for your work!*

At Rutgers the hard-working student  
Sleeps only when sitting in class.  
All night the same diligent student  
Stays awake thinking hard how to pass.

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\* A problem like this was given on an honors calc exam at Stanford. There the slug's average height was computed using a surface integral, an idea we will discuss soon. The method above will yield the same answer.