Differentiation

Definition:

$f'(a) = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

General formulas:

Product: $(uv)' = u'v + uv'$

Quotient: $(u/v)' = [u'v - uv']/v^2$

Chain rule: $[f(u)]' = f'(u) \cdot u'$

Constant multiple: $(cu)' = cu'$

Inverse: $dx/dy = 1/(dy/dx)$

Special functions:

Constants: $c' = 0$

Powers: $\dfrac{d}{dx}x^n = nx^{n-1}$

Exponential, logarithmic:

$\dfrac{d}{dx}e^x = e^x$; $\quad \dfrac{d}{dx}\ln(x) = 1/x$

$\dfrac{d}{dx}a^x = \ln a \cdot a^x$

Trigonometric:

$\dfrac{d}{dx}\sin(x) = \cos(x)$; $\quad \dfrac{d}{dx}\cos(x) = -\sin(x)$

$\dfrac{d}{dx}\sec(x) = \sec(x)\tan(x)$; $\dfrac{d}{dx}\csc(x) = -\csc(x)\cot(x)$

$\dfrac{d}{dx}\tan(x) = \sec^2(x)$; $\dfrac{d}{dx}\cot(x) = -\csc^2(x)$

$\dfrac{d}{dx}\sin^{-1}(x) = \dfrac{1}{\sqrt{1-x^2}}$; $\dfrac{d}{dx}\cos^{-1}(x) = \dfrac{-1}{\sqrt{1-x^2}}$

$\dfrac{d}{dx}\tan^{-1}(x) = \dfrac{1}{1+x^2}$; $\quad \dfrac{d}{dx}\cot^{-1}(x) = \dfrac{-1}{1+x^2}$

$\dfrac{d}{dx}\sec^{-1}(x) = \dfrac{1}{x\sqrt{x^2-1}}$

$\dfrac{d}{dx}\csc^{-1}(x) = \dfrac{-1}{x\sqrt{x^2-1}}$

Mean Value Theorem:

$f(x)$ cont. on $[a,b]$, diff. on (a,b): one can solve

$$f'(x) = \frac{f(b)-f(a)}{b-a},$$

with $a < x < b$.

Graphing

Symmetry:

Even: $f(-x) = f(x)$ (symmetric)

Odd: $f(-x) = -f(x)$ (skew symmetric)

Asymptotes:

Horizontal: $\lim_{x\to\pm\infty} f(x) = a$ (two cases)

Vertical: $\lim_{x\to a^\pm} f(x) = \pm\infty$ (four cases)

Increasing/Decreasing:

f' Positive on interval: Increasing

f' Negative on interval: Decreasing

Local maxima/minima:

Critical numbers: $f'(x) = 0$, or undefined;

or Endpoints

Concavity, inflection:

$f''(x) > 0$: upward; $\quad f''(x) < 0$: downward

$f''(x)$ *changes sign*: Inflection ($\sim$)

Differentials and Newton's method

$dy = y'dx$; $y \approx y_0 + dy$

$f(a+\Delta x) \approx f(a) + f'(a)\Delta x$

Newton's method: $x^{\text{new}} = x - f(x)/f'(x)$ (iterate)

Intermediate Value Theorem:

If $f(x)$ is continuous on $[a,b]$, $f(a) < N < f(b)$, then the equation $f(x) = N$ is solvable, with $a < x < b$.

Limits

L'Hospital: $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}$

– (when applicable)

$\lim_{n\to\infty}(1+1/n)^n = \lim_{h\to 0}(1+h)^{1/h} = e$

Squeeze Theorem:

If $g_1(x) \le f(x) \le g_2(x)$ near a,
and $\lim_{x\to a} g_1(x) = \lim_{x\to a} g_2(x) = L$,
then $\lim_{x\to a} f(x) = L$.

Example: $\lim_{x\to 0} \dfrac{\sin(x)}{x} = 1$

Interpretations of derivatives

1st: Velocity or rate of change

2nd: Acceleration

Logarithmic: Relative rate of change

Slope of tangent line

Integration

Integration gives the *signed area* between the curve and the x-axis (above−below).

Fundamental Theorem of Calculus:

(f *continuous*:) $\int_a^b f(x)dx$ is $F(b) - F(a)$, with $F(x)$ an antiderivative.

Formulas:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \;(n \neq -1)$$

$$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$$

Read the **differentiation formulas** from right to left!

Algebra

Slope: $\Delta y/\Delta x$; *Distance*: $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

Quadratic formula: $ax^2 + bx + c = 0$: then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$(a-b)(a+b) = a^2 - b^2$

$\ln(a^b) = b\ln(a)$

$\ln(ab) = \ln(a) + \ln(b)$; $\ln(1/b) = -\ln(b)$

$\ln(1) = 0$; $\ln(e) = 1$

$\log_a(x) = \dfrac{\ln(x)}{\ln(a)}$

Area:

Triangle: 1/2 base×altitude

Circle: πr^2; *Sphere (surface)*: $4\pi r^2$

Volume:

Box: Product of dimensions.

Sphere (inside): $\frac{4}{3}\pi r^3$

Cylinder: Base area × Height

Cone: $\frac{1}{3}$ Base area × Height

Trigonometry

Values:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Right Triangles:

sine: opposite/hypotenuse

cosine: adjacent/hypotenuse

tangent: opposite/adjacent

secant: 1/cosine

Multiples:

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin^2(x/2) = \dfrac{1-\cos(x)}{2}$; $\cos^2(x/2) = \dfrac{1+\cos(x)}{2}$

More identities:

$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$

$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

Co-functions:

$\cos(x) = \sin(\pi/2 - x)$; $\cot(x) = \tan(\pi/2 - x)$

$\csc(x) = \sec(\pi/2 - x)$

Numbers (*rough approximations*)

$\pi \approx 3.14$ $e \approx 2.7$ $\sqrt{2} \approx 1.4$ $\sqrt{3} \approx 1.7$

$\ln 2 \approx .7$ $\ln 10 \approx 2.3$