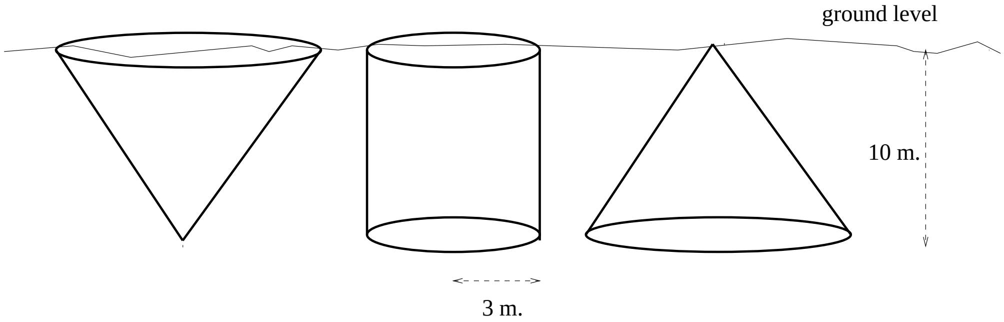
Problems, Week 2

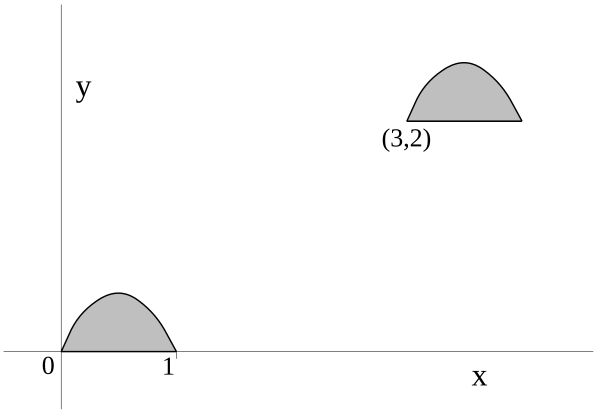
1. Toxic waste (a homogeneous liquid whose density is 300 kg/m^3) is stored in three buried containers. Each container, illustrated below, is 10 meters tall. The top of each container is at ground level. Each container has the *same* volume. The middle container is a cylinder, and the other two are circular cones. Which container needs the *least* amount of work to empty (that is, to pump the toxic waste to ground level)? Which container needs the *most* work to empty? Justify your assertions by computing the work necessary in each case*.



Toxic waste containers

Note: the volume of a cylinder with base radius r and height h is $\pi r^2 h$; the volume of a cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.

2. a) Suppose the area enclosed by $y = 2x(1-x)$ and the $x-$axis is moved so that its left corner is at $(3,2)$. Compute the volume obtained when this translated area is revolved, first about the $x-$axis and then about the $y-$axis.



b) Is it possible to move the area specified in a) so that its left corner is at a point (x_0, y_0) in the first quadrant *and* so that the volumes resulting when the area is revolved about both of the axes are equal? Explain your answer.

3. If a freely falling body starts from rest, then its displacement is given by $s = \frac{1}{2}gt^2$. Let the velocity after a time T be v_T. Show that if we compute the average of the velocities

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* You probably should begin with the cylinder.

with respect to t we get $v_{\text{ave}} = \frac{1}{2}v_T$, but if we compute the average of the velocities with respect to s we get $v_{\text{ave}} = \frac{2}{3}v_T$.

4. A sort of raindrop is obtained by revolving the profile curve

$$y = \sqrt{x}(x - C)^2 \text{ for } 0 \leq x \leq C$$

about the $x-$axis. Here C is a positive constant.

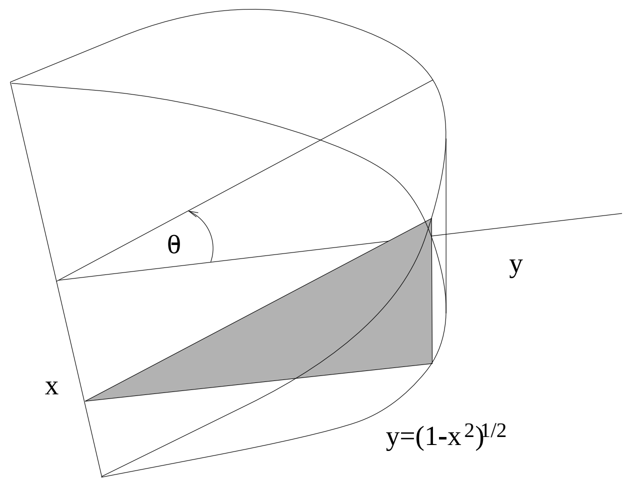
a) Sketch the profile curve and the solid of revolution.

b) For which value of C will the raindrop have volume 1? What are the rough dimensions (length and diameter) of this raindrop?

5. A wedge is cut from a cylindrical tree of radius 1. One cut is parallel to the ground. The other cut is at an angle θ with the ground, and the two cuts meet at the central axis of the tree.

a) What is the area of the shaded triangular slice in terms of θ and x?

b) What is the volume of the wedge?



6. Electrons repel each other with a force which is inversely proportional to the square of the distance between them; call the proportionality constant k in the units to be used. Suppose one electron is fixed at $x = 0$ on the $x-$axis.

a) Find the work done in moving a second electron along the $x-$axis from the point $x = 10$ to the point $x = 1$.

b) Find the work done in moving the second electron along the $x-$axis from the point $x = M$ to the point $x = 1$.

c) What happens to your answer in b) (which should depend on M) as $M \to +\infty$?

7. Electrons repel each other with a force which is inversely proportional to the square of the distance between them; call the proportionality constant k in the units to be used. Two electrons are held fixed on the $x-$axis at the points $x = 1$ and $x = -1$. Find the work done in moving a third electron along the $x-$axis from the point $x = 10$ to the point $x = 2$.