Problems, Week 1

1. Suppose that Q is the function

$$Q(x) = \left(\arctan\Big(\ln\big(\sqrt{x} - 1\big)\Big)\right)^3 .$$

a) What are the domain and range of Q? Answers should *not* be numerical approximations, but should be written if needed in terms of constants studied in calculus such as π and e.

b) Suppose $y = Q(x)$. Write a formula for x in terms of y.

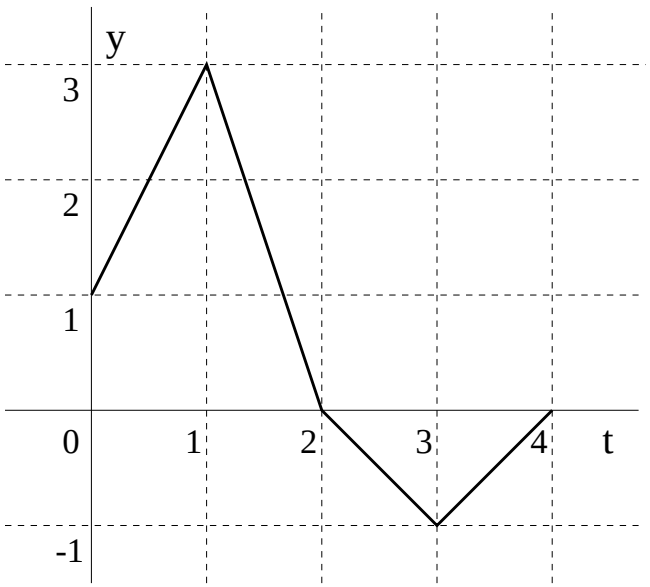
2. A radioactive substance A decays at a rate proportional to the amount of the substance present.

a) Suppose that an initial amount of 10 micrograms decays after 8 hours to 7 micrograms. Determine a formula for $A(t)$, the amount of substance A present at time t.

b) In the presence of a certain gamma ray flux, the radioactive decay of the substance is increased. In fact, when an initial amount of 10 micrograms of A is subject to this radiation, after 8 hours only 2 micrograms of A remain. Determine a formula for $B(t)$, the amount of substance A present at time t when the radiation mentioned is present.

c) Suppose we are presented with 10 micrograms of substance A and wish to have 5 micrograms after 8 hours. We are allowed to "turn on" the gamma radiation at some time during the 8 hours (but it must stay on after it is turned on!). At what time should the radiation be introduced in order to obtain 5 micrograms of A after 8 hours?

3. Suppose the graph of f is given by:



The graph of $y = f(t)$

Let $g(x) = \int_0^x f(t)\,dt$ for $0 \leq x \leq 4$.

a) Find $g(0)$, $g(2)$, and $g(4)$.

b) Find all values of x for which $g(x)$ is increasing, and explain your answer. Find all values of x for which $g(x)$ is decreasing, and explain your answer.

c) What is the maximum value of $g(x)$?

4. a) Find the number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$, $1 \leq x \leq 4$.

b) Find the number b such that the line $y = b$ bisects the area in part a).