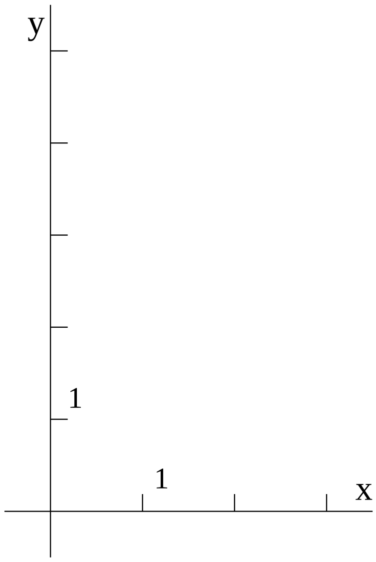
1 A

(20) 1. a) Verify using calculus that $\displaystyle\int_1^e \frac{(\ln x)^2}{x}\,dx = \frac{1}{3}$.

b) Verify using calculus that $\displaystyle\int_1^e x^2 \ln x\; dx = \frac{2}{9}e^3 + \frac{1}{9}$.

(16) 2. a) Sketch the region in the first quadrant which is bounded by the curves $y = x^2$ and $y = 2x(3 - x)$. Label the curves.



b) Set up and compute a definite integral for the area of the region sketched in a). Compute this integral using calculus.

c) Set up a definite integral for the volume obtained when the region sketched in a) is revolved about the $x-$axis. Describe the method you are using. Do **not** evaluate the integral.

3 A

(16) 3. a) Verify using calculus that $\displaystyle\int_0^{\pi/2} (\cos x)^3 (\sin x)^2 \; dx = \frac{2}{15}.$

b) Verify using calculus that $\displaystyle\int_0^{1/2} \sqrt{1-x^2}\; dx = \frac{\sqrt{3}}{8} + \frac{\pi}{12}.$

(12) 4. Find an equation of the curve that passes through the point $(1, 2)$ and whose slope at each point (x, y) on the curve is $x^3 y^2$.

(12) 5. a) Find the indicated indefinite integral: $\displaystyle\int \frac{x}{(x+2)(4-x)}\,dx.$

b) Use your answer to a) and properties of the logarithm function to verify that

$$\int_0^2 \frac{x}{(x+2)(4-x)}\,dx = \frac{1}{3}\ln 2$$

(10) 6. Use an improper integral to find an integer, N, so that the partial sum $S_N = \sum_{n=1}^{N} \frac{1}{n^3}$ is within 10^{-7} of the sum of the whole infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. Be sure to explain why the value of N you give is a correct answer. Do **not** calculate S_N.

7 A

(15) 7. a) Use integration by parts twice to compute $\int (\sin x)\left(e^{-x}\right) dx$.

b) Use your answer to a) to verify that $\int_0^{\infty} (\sin x)\left(e^{-x}\right) dx$ converges and is equal to $\frac{1}{2}$.

(12) 8. We consider here numerical approximation of the definite integral $\int_0^1 \cos\left(x^2\right)\,dx$.

a) Write out and compute the Trapezoid Rule approximation to this definite integral with $n = 4$ subdivisions. You may use the table of values of $\cos\left(x^2\right)$ given below. Show the formula you use and give a numerical answer.

x	$\cos\left(x^2\right)$
0.0	1.00000
0.25	.99805
0.5	.96891
0.75	.84592
1.0	.54030

b) Find n (the number of subdivisions) so that the Trapezoidal Rule estimate will be within 10^{-6} of the true value of the definite integral. You may use the error bound $\dfrac{K(b-a)^3}{12n^2}$ where K is an overestimate of the magnitude of the second derivative. Do **not** calculate this Trapezoidal Rule approximation.

(15) 9. We consider here another approach to the numerical approximation of the definite integral $\int_0^1 \cos\left(x^2\right)\,dx$.

a) Write a power series centered at 0 for $\cos x$.

b) Use your answer to a) to write a power series centered at 0 for $\cos\left(x^2\right)$.

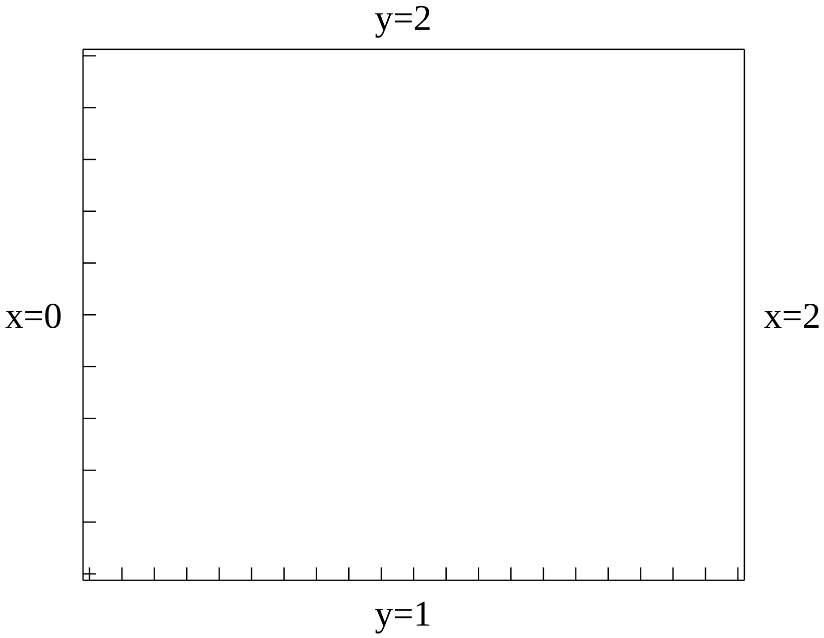
c) Use your answer to b) to write the definite integral as a series. Be sure to give a **formula** for the general term of this series.

d) Write a specific partial sum of this series obtained in c) which approximates the definite integral with an error of at most 10^{-6}. Be sure to explain how you know your answer is correct. Do **not** calculate this partial sum.

(18) 10. Suppose $f(x) = (1+x)^{1/2}$.

a) Find $T_2(x)$, the second degree Taylor polynomial for $f(x)$ centered at $x = 0$.

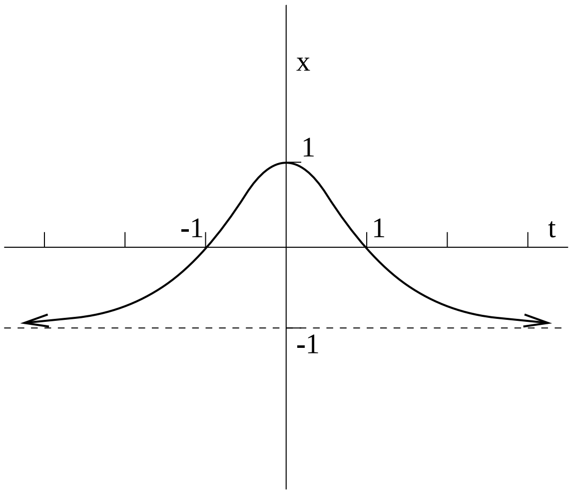
b) Sketch a graph on the same coordinate axes of both $f(x)$ and $T_2(x)$ in the window $0 \le x \le 2$ and $1 \le y \le 2$. Label the curves you sketch. Marks have been drawn on both the vertical and horizontal sides of the window below at intervals of .1.



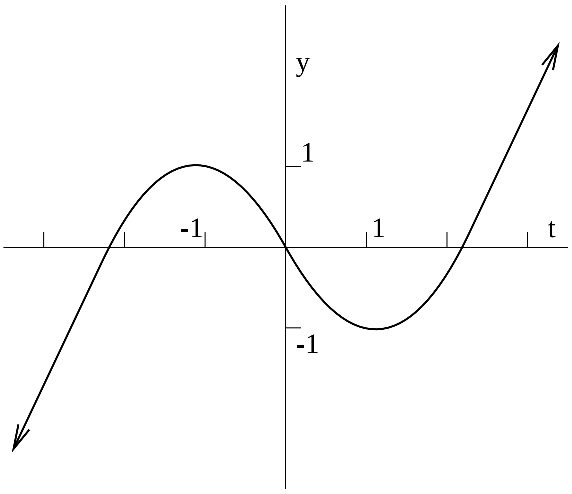
c) Use your graph to find a positive number A so that $|f(x) - T_2(x)| < .1$ for all x in the interval $[0, A]$. Be sure to explain your answer by referring to the graph you have drawn.

d) Use the remainder formula in Taylor's Theorem to find a positive number B so that $|f(x) - T_2(x)| < 10^{-10}$ for all x in the interval $[0, B]$. Be sure to explain how you know that your answer is correct.

(16) 11. A curve is described by parametric equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ where the graphs of x as a function of t and y as a function of t are given below.



x=f(t)



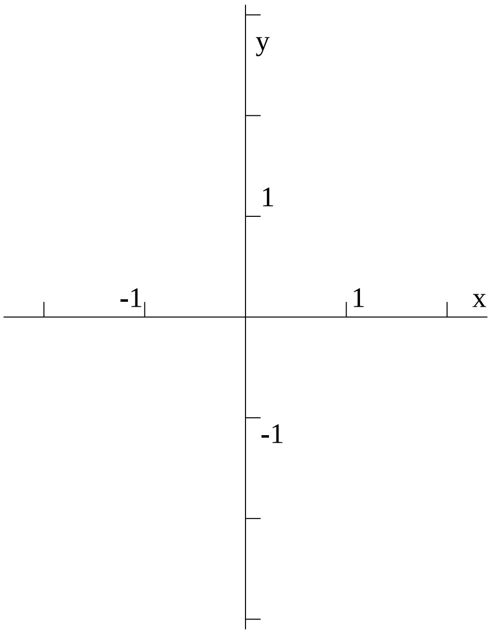
y=g(t)

a) What is the position of the point on the curve $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ when $t = 0$? ______

b) For which value(s) of t is the tangent line to the curve $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ horizontal? ______

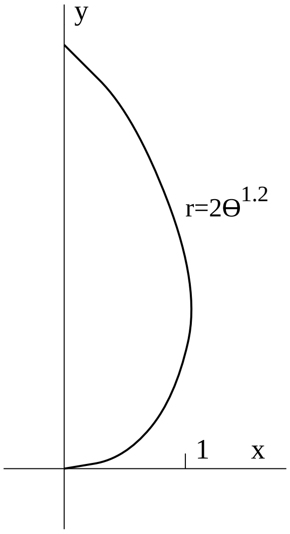
c) For which value(s) of t is the tangent line to the curve $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ vertical? ______

d) Sketch the curve $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ using all the information given.



d) As $t \to +\infty$, which line (if any) does the curve $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ approach? ______

(10) 12. A graph of part of the polar coordinate curve $r = 2\theta^{1.2}$ for $0 \le \theta \le \dfrac{\pi}{2}$ is shown below.



Use calculus to find the area enclosed by the $y-$axis and the part of the polar coordinate curve shown. You may leave the answer in unsimplified numerical form.

(14) 13. a) Determine all values of x for which $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n2^n}$ converges absolutely.

b) Determine all values of x for which $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n2^n}$ converges conditionally.

(14) 14. a) What is $T_n(x)$, the n^{th} degree Taylor polynomial for e^x centered at $x = 0$?

b) Use the remainder formula in Taylor's Theorem to find an integer n so that

$$|T_n(x) - e^x| < .1$$

for all x in the interval $[-2, 2]$. Be sure to explain how you know that your answer is correct.

A A

FINAL EXAM for MATH 152 and 154

May 1, 1996

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

Please write the name of your instructor, your course number, and your section number:

Instructor ______________ **Course** ________ **Section** ________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use <u>one</u> sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use <u>no</u> other materials.

Problem Number	Possible Points	Points Earned:
1	20	
2	16	
3	16	
4	12	
5	12	
6	10	
7	15	
8	12	
9	15	
10	18	
11	16	
12	10	
13	14	
14	14	
Total Points Earned:		

A A