Problems, Week 5

1. The curve $y = ax^2 + bx + c$ passes through the point $(1, 2)$ and is tangent to the line $y = x$ at the origin. Find a, b, and c.

2. Consider the curves $y = x^2$ and $y = C - x^2$ (here C is a constant to be precisely determined later).

a) Sketch the curves $y = x^2$ and $y = C - x^2$ for $C = -1$, $C = 1/2$, and $C = 2$ on the same coordinate axes.

b) Find a value of C so that the curves $y = x^2$ and $y = C - x^2$ have two intersection points, and so that, at the intersection points, the lines tangent to the two curves are perpendicular. Sketch the curves and the tangent lines for this particular value of C. (Note: such pairs of curves are very important in the study of both electromagnetism and heat transfer. They are called "orthogonal" curves.)

3. At time $t = 0$, Irwin fires a rubber band at a spider on the ceiling. At time $t = 3$, it hits the mark. Let $s(t)$ represent the height of the rubber band above the floor at time t.

a) Which is larger, $s(1)$ or $s(2)$?

b) Which is larger, $s'(1)$ or $s'(2)$?

c) Does your answer in part a) or b) depend on what units you use for s or t (for instance, feet vs. meters, or seconds vs. minutes)?

4. Is it possible for two differentiable functions f and g to satisfy all of the following conditions at once: $f(0) = 0$, $g(0) = 0$, and $f(x)g(x) = x$ for all x?

5. a) If $f(x) = u(x)v(x)w(x)$, find a formula for $f'(x)$ in terms of $u(x)$, $v(x)$, $w(x)$, $u'(x)$, $v'(x)$, and $w'(x)$.

b) Find $g'(x)$ if $g(x) = (x^2 + 1)(2x^3 - x + 4)(x^{12} + x^6 + 5)$.

6. Consider the curve $y = x^3 + 3x^2 - 3x$.

a) This curve has two tangent lines which are parallel to $y = 6x - 8$. Where are the points of tangency of these two tangent lines? (Drawing a picture might help.)

b) There is a value m so that the curve has precisely one tangent line whose slope is m. What is that value of m, and where is the point of tangency?