Corrections to

Discrete Fourier and Wavelet Transforms:
An Introduction through Linear Algebra
with Applications to Signal Processing

by Roe W. Goodman

Revised October 15, 2017

A negative line number means measured from the bottom, not including footnotes.

p. 9 l. -6: change $u_2 = \begin{bmatrix} 0 & 1 & -3 \end{bmatrix}$ to $u_2 = \begin{bmatrix} 0 & 1 & -3 \end{bmatrix}$

p. 11 l. 1 of §1.5.1: change $v_m$ to $v_n$

p. 23 l. 5 right side of equation: change to $2 |\alpha|^2 |f[n]|^2 + 2 |\beta|^2 |g[n]|^2$

p. 26 l. 9: change $u$ to $w$ (twice)

p. 43 l. 3: add The highs and lows of the fundamental 3 Hertz sine wave are modified to include ripples generated by the 9 Hertz sine wave.

p. 46, line after equation (2.7): change For $N = 4$ we have $w = e^{2\pi i / 4} = i$ to For $N = 4$ we have $\omega = e^{2\pi i / 4} = i$

p. 46 l. -4: change right side to $N[d_0 d_1 \ldots d_{N-1}]^T$

p. 47 equation (2.10): change middle term to $N(d_0 E_0 + d_1 E_1 + \cdots + d_{N-1} E_{N-1})$

p. 50 l. -8: change so it the same to so it is the same

p. 55 4th line of Remark 2.3: change $y[k]$ to $\hat{y}[k]$

p. 57 l. -1: add sentence This means that $T$ removes most of the high-frequency energy ($\theta \approx \pi$) in the input signal but does not significantly change the energy in the low frequencies ($\theta \approx 0 \mod 2\pi$).

p. 57 l. -3: change $+i(1 - c^2)/4$ to $-i(1 - c^2)/4$

p. 57 l. -1: change $z[k] = 2c^2 \sin(3k\pi/4) + \frac{1-c^2}{2} \sin(9k\pi/4)$ to $z[k] = 2c^2 \sin(k\pi/4) + \frac{1-c^2}{2} \sin(3k\pi/4)$

p. 58 l. 2: change $2c^2 \sin(3k\pi/4)$ to $2c^2 \sin(k\pi/4)$

change $\frac{1-c^2}{2} \sin(9k\pi/4)$ to $\frac{1-c^2}{2} \sin(3k\pi/4)$

p. 58 Example 2.10, l. 5: change 4,092 to 4,096

p. 61 equation (2.31): change the index range to $j = 0, \ldots, m - 1$

p. 63 lines 3,4 of (b): change to

...can be downloaded or added as a MATLAB toolbox from
http://www.mathworks.com/moler/chapters

p. 71 (3)(c): CHANGE Definition 2.27 TO Definition 2.5

p. 77 l. -1: AFTER DISPLAYED FORMULA INSERT For example, when $N = 4$ then

$$\text{split} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$ 

In this case split happens to be a symmetric matrix, and so \text{merge} is the same matrix. This is not the case for larger $N$, however.

p. 79 l. 10: INSERT THE FOLLOWING FOOTNOTE The terms prediction, update, and normalization were introduced in [Daubechies and Sweldens (1998)] to describe the basic steps in the lifting method for wavelet transforms. This paper, subtitled Research Tutorial, is very readable and explains the reasons for this terminology.

p. 79 l. 6: INSERT In this calculation we have used the fact that \text{split} is an orthogonal matrix.

p. 80, equation (3.3): CHANGE $T_s^{(k)} = \begin{bmatrix} I^{(k-1)} & I^{(k-1)} \\ I^{(k-1)} & -I^{(k-1)} \end{bmatrix} \text{split}$ TO $T_s^{(k)} = \begin{bmatrix} I^{(k-1)} & I^{(k-1)} \\ I^{(k-1)} & -I^{(k-1)} \end{bmatrix} \text{merge}$

p. 90, formula (3.24): CHANGE $P = \begin{bmatrix} I & 0 \\ -(1/4)(\sqrt{3}I - (\sqrt{3} - 2)S) & I \end{bmatrix}$ TO $P = \begin{bmatrix} I & 0 \\ -(1/4)(\sqrt{3}I + (\sqrt{3} - 2)S) & I \end{bmatrix}$.

p. 91 3rd line of the proof of Theorem 3.1: CHANGE THE MATRIX ON THE RIGHT TO

$$\begin{bmatrix} (aI + cS) & -(bI + dS^{-1}) \\ (bI + dS) & (aI + cS^{-1}) \end{bmatrix}$$

p. 91 4th line of the proof of Theorem 3.1: CHANGE $\text{split}$ is orthogonal TO $\text{split}$ is orthogonal and $S^{-1} = S^T$.

p. 94 l. -2, second row of matrix: CHANGE $z$ TO $z^{-1}$ (TWO CHANGES).

p. 95 l. -7: ADD MISSING LEFT PARENTHESES TO GET

$$d^{(1)}[n] = x_{\text{odd}}[n] - ((9/8)I + (3/8)S^{-1})s^{(1)}$$

p. 99 l. -4: CHANGE Section 2.4 TO Section 2.5

p. 100 l. 3: CHANGE

$$\mathbf{u} = [u[N-1], \ldots, u[1], u[0]] \text{ if } \mathbf{u} = [u[0], u[1], \ldots, u[N-1]].$$
Thus when $\mathbf{u}$ and $\mathbf{\check{u}}$ are viewed as $N$-periodic functions on the integers, then $\mathbf{\check{u}}[k] = \mathbf{u}[-k]$.

p. 100 l. 11: CHANGE $= (\mathbf{u}_0 \ast \mathbf{x}[k])[2j] = (\mathbf{u}_0 \ast \mathbf{x}[k])_{\text{even}}[j]$ TO $= (\mathbf{u}_0 \ast \mathbf{x})[2j] = (\mathbf{u}_0 \ast \mathbf{x})_{\text{even}}[j]$.

p. 102 l. 16: CHANGE $\mathbf{x}_s$ and $\mathbf{x}_d$ TO $\mathbf{x}_s$ and $\mathbf{x}_d$.

p. 105 (3.47): In the formulas for $\mathbf{U}_3$ and $\mathbf{V}_3$ each $\frac{1}{2}$ should be $\frac{1}{8}$ and in the formula for $\mathbf{V}_2$ each $\frac{1}{2}$ should be $\frac{1}{4}$.

p. 106 l. -5: CHANGE $S_{(k-1)}$ TO $S_{(j-1)}$.

p. 119 l. -9: BEFORE We define the mean square error
INSERT
For a matrix $\mathbf{A} = [a_{ij}]$ we define the norm (more precisely, the Frobenius norm) of $\mathbf{A}$ to be $\|\mathbf{A}\| = \left( \sum_{i,j} |a_{ij}|^2 \right)^{1/2}$. When $\mathbf{A}$ is a row vector or a column vector, this is the same definition as in Section 1.7. In general, the Frobenius norm of a matrix $\mathbf{A}$ of size $M \times N$ is the same as the norm of the column vector $\mathbf{u}$ of size $MN \times 1$ obtained by concatenating the columns of $\mathbf{A}$. In MATLAB $\|\mathbf{A}\|$ is calculated by the command `norm(A, 'fro')`.

p. 119, l. -5, -4, -3: CHANGE
for the round-wavy image we calculate that MSE = 0.008, while for the kitten image MSE = 0.695. The MSE for the compressed kitten image is about 85 times larger than the MSE for the compressed synthetic image
TO
for the round-wavy image we calculate that MSE = 0.222, while for the kitten image MSE = 17.1. The MSE for the compressed kitten image is about 77 times larger than the MSE for the compressed synthetic image

p. 120, l. 1: CHANGE MSE = $1/128 = 0.008$ TO MSE = 1.

p. 120, l. 8, 9: CHANGE
For the round-wavy image we calculate that PSNR = 69.0, while for the kitten image PSNR = 49.7.
TO
For the round-wavy image we calculate that PSNR = 54.7, while for the kitten image PSNR = 35.8.

p. 123 l. 20: CHANGE You should get the same matrix as in Example 3.12 TO You should get the analysis matrix in Example 3.12 multiplied by $1/(4\sqrt{2}) = 0.1767\ldots$

p. 123 l. -11: CHANGE From (3.58) TO From (3.19).

p. 124 lines 18, 19: CHANGE
Ts = cdfsmat(8), norm(Ts*Ta - eye(8))
You should get the same matrix as in Example 3.12
TO
Ts = cdfsmat(8), norm(Ts*Ta - eye(8), 'fro'))
The matrix Ts should be the synthesis matrix in Example 3.12 multiplied by $4\sqrt{2}$, and the norm value should be (essentially) zero.

p. 126 lines 2-4: CHANGE
Check this property by setting Ts = Ta' and calculating the distance

$$\text{norm}(Ta*Ta' - eye(8))$$

between $T_a T_a^T$ and the identity matrix.
TO
Check this property by calculating

$$\text{norm}(Ta*Ta' - eye(8), 'fro'))$$

(the distance between $T_a T_a^T$ and the identity matrix). This norm value should be (essentially) zero.

p. 126 line -12: CHANGE $\text{norm}(Wa*Wa' - eye(8))$
TO $\text{norm}(Wa*Wa' - eye(8), 'fro'))$

p. 130 Section 3.7.5 (a) l. 9 of the m-file: MOVE
s1shift = [s1(N/2)
TO BEGINNING OF NEXT LINE
p. 130 Section 3.7.5 (a) l. 12 of the m-file: end the line with semicolon ;

p. 130 Section 3.7.5 (a) Next to last line of the m-file: MOVE
d = (sqrt(3)+1)/sqrt(2)*d1;
TO NEW LINE

p. 138 Exercise 7 (b): CHANGE EQUATION TO

$$\begin{bmatrix} s(t) \\ d(t) \end{bmatrix} = U \begin{bmatrix} x_{\text{even}} \\ d(1) \end{bmatrix}$$

p. 138 (9) l. 2: CHANGE equations (3.42)
TO equations (3.42) with $S_8$ replaced by $S_N$ and $k = 0, \ldots, N - 1$.

p. 147 l. 4: CHANGE $Tx$ TO $Hx$

p. 148, Equation (4.10) : CHANGE $\langle x, H^T * y \rangle$ TO $\langle x, H^T y \rangle$

p. 152, Proposition 4.1: CHANGE
Let $u = [h[0] \ h[1] \cdots \ h[L-1] \ 0 \cdots 0]$ be the $1 \times M$ row vector consisting of the filter coefficients padded by zeros.
TO
Let $u = [h[0] \ 0 \cdots 0 \ h[L-1] \ h[L-2] \cdots \ h[1]]$ be the $1 \times M$ row vector consisting of the shifted and reversed filter coefficients padded by zeros as indicated.
p. 152, proof of Proposition 4.1: change
This follows immediately from (4.19), as in the proof of Theorem 3.2.

To
From formula (4.19) we check that \( TS^2x = STx \) for all signals \( x \) (the change from \( S^2 \) to \( S \) in this equation comes from downsampling). Hence

\[
U(SM)^2P_Mx = UP_MS^2x = P_{M/2}TS^2x = P_{M/2}STx = SM/2UP_Mx,
\]

where we have also used formula (4.18) in the first equality, formula (4.20) in the second equality, and formulas (4.18) together with (4.20) in the last equality. Since this relation holds for all \( M \)-periodic signals \( x \), we conclude that \( U(SM)^2 = SM/2U \). Just as in the proof of Theorem 3.2, this equation implies that each row of \( U \) below the first row is obtained by multiplying the row above it on the right by \( (SM)^{-2} \) (which shifts the row to the right two positions with wraparound).

To determine the first row of \( U \), consider the \( M \)-periodic signals \( x_p = (\delta_p)_p,M \) for \( p = 0,1,\ldots,M-1 \). Then \( P_Mx_p \) is the standard basis vector \( e_{p+1} \) in \( \mathbb{R}^M \). Hence \( T_{x_p}[k] \) is the entry in row \( k+1 \) and column \( p+1 \) of \( U \) for \( k = 0,\ldots,M/2-1 \). From formula (4.19)

\[
T_{x_p}[0] = \sum_{j=0}^{M-1} h[j] x_p[-j] = h[M-p] \quad \text{for } 0 \leq p \leq M-1.
\]

Thus the first row of \( U \) is \( u = [h[0] \ h[M-1] \ h[M-2] \ \cdots \ h[1]] \). Since \( h[k] = 0 \) for \( k \geq L \), this proves that the matrix \( U \) has the form given in the proposition.

p. 152, Example 4.6: change

\[
U = \begin{bmatrix}
\end{bmatrix}.
\]

Notice the wrap-around that occurs on the last row.

To

\[
U = \begin{bmatrix}
\end{bmatrix}.
\]

Notice the wrap-around in the second, third, and fourth rows..

p. 152 l. 4 of Theorem 4.4: change \( \omega_N = e^{2\pi ki/N} \) to \( \omega_N = e^{2\pi i/N} \)

p. 154 l. 9: add sentence This is an arrangement of filters that splits a signal into subsignals using downsampling and then combines the subsignals using upsampling. The downsampling and upsampling make this an efficient way to analyze a signal.

p. 168 l. 4: change \( H_1(z) = \frac{\sqrt{2}}{4}(z^2 - 2 + z^{-1}) = \frac{\sqrt{2}}{2}(\cos(\omega) - 1) \)
TO $H_1(z) = \frac{z\sqrt{2}}{4}(2 - z - z^{-1}) = \frac{z\sqrt{2}}{2}(1 - \cos(\omega))$

p. 170 l. 4: CHANGE $= x^2(1 + 3y) + y^2(1 + 3x)$
TO $= x^2(x + 3y) + y^2(y + 3x)$

p. 170 l. 6: CHANGE $= x^3(1 + 5xy + 10y^2) + y^3(1 + 5xy + 10x^2)$
TO $= x^3(x^2 + 5xy + 10y^2) + y^3(10x^2 + 5xy + y^2)$

p. 172, Example 4.16: CHANGE $g_0 = \frac{\sqrt{2}}{8} (\delta_{-3} + 3\delta_{-2} + 3\delta_1 + \delta_0)$
TO $g_0 = \frac{\sqrt{2}}{8} (\delta_{-3} + 3\delta_{-2} + 3\delta_{-1} + \delta_0)$

p. 175 last line of Section 4.5: CHANGE Exercise 4.12 #13 TO Exercise 4.12 #11

p. 176 l. -17: CHANGE two polynomials in the first row
TO two polynomials in the first column

p. 176 l. -4: CHANGE Its determinant is $-2z \det H_p(z)$.
TO Its determinant is $-2z \det H_p(z^2)$.

p. 179 Theorem 4.10 ADD where $g(z)$ and $h(z)$ are Laurent polynomials.

p. 181 right side of Equation (4.70): CHANGE MATRIX ENTRIES $(b(z) - b(z)g(z))$
TO $(b(z) - d(z)g(z))$

p. 189 l. -12: CHANGE $z^{2K-1}H_1(-z) = H_1(z^{-1})$ TO $z^{2K-1}H_0(-z) = H_1(z^{-1})$

p. 190 Equation (4.86): CHANGE THE TERM $-h_0[1] z^{2K-2}$ TO $-h_0[1] z^{-2K+2}$

p. 192 l. 6: The right side should be $-\frac{1}{2} \ldots$ (with a small space after the minus sign)

p. 192 l. -1: CHANGE TO

$$4\sqrt{2}H_1(z) = -b - dz^{-2} + a + c z^{-2} = a z - b + c z^{-1} - dz^{-2} = 4\sqrt{2}z^{-2}H_1(z^{-1})$$

p. 193 l. -6: CHANGE $z^{-1}(z - r) = r(1 - rz^{-1})$ TO $z^{-1}(z - r) = (1 - rz^{-1})$

p. 195 Section 4.11.1 end of line 10 and line 11 of (a): CHANGE +1.5 TO +1

p. 197 Section 4.11.2 (a): END THE FIRST LINE OF CODE WITH SEMICOLON ;

p. 201 Section 4.11.3 (c): CHANGE measured by the Mean Square Error (MSE):

$$\text{MSE} = (\text{norm}(X1 - X2)^2)/2^{16}$$

TO measured by the Mean Square Error (MSE):

$$\text{MSE} = (\text{norm}(X1 - X2, 'fro')^2)/2^{16}$$
(see Section 3.6.4).

p. 203 Section 4.11.4 (a): change Calculate $\|X\|$ and $\|Y\|$.
  TO Calculate $\|X, \text{ 'fro'}\|$ and $\|Y, \text{ 'fro'}\|$.

p. 204 l. 9: change idea conditions TO ideal conditions

p. 204 l. 15, 16: change random normal integers (with mean zero, standard deviation 50)
  TO random integers (the integer parts of independent normal random variables with mean
  zero, standard deviation 50)

p. 213 last line of Proof. (1): change (verification left as an exercise)
  TO (verification left to Exercise 5.7 #5)

p. 222 ll. 11, 12: change $2^{j/2}$ TO $2^{-j/2}$

p. 234 l. -9: change From this equation
  TO From this equation and formula (5.28) for the operators $M_\alpha$

p. 234 ll. -5, -6: change

\[
\langle S^n \phi_\alpha, D\phi \rangle = \sum_{k\in\mathbb{Z}} g_\alpha[k] \langle DS^{2n+k} \phi, D\phi \rangle = g_\alpha[-2n] = h_\alpha[2n],
\]

\[
\langle S^n \phi_\alpha, DS\phi \rangle = \sum_{k\in\mathbb{Z}} g_\alpha[k] \langle DS^{2n+k-1} \phi, D\phi \rangle = g_\alpha[-2n + 1] = h_\alpha[2n - 1].
\]

TO

\[
\langle S^n \phi_\alpha, D\phi \rangle = \sum_{k\in\mathbb{Z}} g_\alpha[k] \langle DS^{2n+k} \phi, D\phi \rangle = \sum_{k\in\mathbb{Z}} g_\alpha[k] \langle S^{2n+k} \phi, \phi \rangle
\]

\[
= g_\alpha[-2n] = h_\alpha[2n],
\]

\[
\langle S^n \phi_\alpha, DS\phi \rangle = \sum_{k\in\mathbb{Z}} g_\alpha[k] \langle DS^{2n+k} \phi, DS\phi \rangle = \sum_{k\in\mathbb{Z}} g_\alpha[k] \langle S^{2n+k-1} \phi, \phi \rangle
\]

\[
= g_\alpha[-2n + 1] = h_\alpha[2n - 1].
\]

p. 234 l. -5: change dilation $D$ preserves inner products
  TO dilation $D$ and shift $S$ preserve inner products

p. 235 l. -8: change is in $V_0$ TO is in $V_0 \oplus W_0$

p. 235 l. -8: change is in $W_0$ TO is in $V_0 \oplus W_0$

p. 237 l. -2: change $\sqrt{(L+1)M}$ TO $\sqrt{(L+1)M}$

p. 241 l. -8: change $H_1(z)$ TO $G_1(z)$

p. 242 l. -3 of Proof of Lemma 5.6: change $g_0$ TO $g_1$

p. 245 l. -3: change $(1024, 1.0), (1024, 1.0), (1548, 1.5)$
TO (512, 0.5), (1024, 1.0), (1536, 1.5)

p. 247 between lines -7 and -8: INSERT \( f(n:2^n-1) = x(n:2^n-1) \);

p. 249 l. -5: CHANGE \( f''(t) = 7t^6 < 7 \) and \( f'''(t) = 42t^5 < 42 \)
TO \( f''(t) = 42t^5 < 42 \) and \( f'''(t) = 210t^4 < 210 \)

p. 249 l. -3: CHANGE \( C(7N^{-2})/(42N^{-3}) = CN/6 = 171C \)
TO \( C(210N^{-2})/(210N^{-3}) = CN/5 = 204C \)

p. 250 l. 6: CHANGE \( f''(t) = 4 \) TO \( f''(t) = -2 \)

p. 250 l. 7: CHANGE \( 4N^{-2} \approx 4C \times 10^{-6} \) TO \( 2N^{-2} \approx 2C \times 10^{-6} \)

p. 251 Exercise #3 (e): CHANGE \( f_1 = f_0 + g_0 + g_1 \) TO \( f_1 = f_0 + g_0 \)

p. 258 II. 4-7 of Section A.3.1: CHANGE click on Desktop Environment, and run the playback files ...
TO Click on Getting started with MATLAB and run the video. Then click on Language Fundamentals. Now click on Basic Matrix Operations, then click on Matrix Manipulation.

p. 258 l. -1: CHANGE \( C \) TO \( B \)

p. 266 Solution (4) (b): CHANGE Theorem 2.29 TO Equation (2.29)

p. 267 Solution (5) (c): CHANGE to \( \lambda_2 = \cdots = 4 + 7\omega^{-2} + 5\omega^{-1} \)

p. 270 Solution (11) (a): CHANGE third row in \( \mathbf{T}_n \) TO \([-2 \ 0 \ 0 \ -2\ 6\ 6]\)

p. 271 Solution (11) (c): CHANGE bottom entry in final formula for \( x_s \) TO 12

p. 272 Solution (15) (d): CHANGE VALUES TO \( \text{MSE} = 0.1875 \) and \( \text{PSNR} = 55.40 \)

p. 274 Solution (4) (b): CHANGE FORMULAS TO
\[ G_0(z) = -z^{-1}H_1(-z) = -z^{-1}(1 + z)(1 - bz) = -z^{-1}(b - 1) + bz \]
\[ g_0 = -\delta_1 + (b - 1)\delta_0 + b\delta_{-1} \]

p. 274 Solution (5) (a): CHANGE FORMULA TO
\[ f(z) = (4 - b) + (4 - 6b + 4c)z^2 + (4c - b)z^4 \]
CHANGE ANSWERS TO
Case i. \( b = 4/5, c = 1/5, \) and \( H_1(z) = (1 - z)(5 + 4z + z^2)/5 \)
Case ii. \( b = 4, c = 1, \) and \( H_1(z) = (1 - z)(1 + 4z + z^2) \)
Case iii. \( b = 4, c = 5, \) and \( H_1(z) = (1 - z)(1 + 4z + 5z^2) \)

p. 274 Solution (5) (b): CHANGE FORMULA TO
\[ G_0(z) = -z^{-1}H_1(-z) \]
CHANGE ANSWERS TO
Case i. \( b = 4/5, c = 1/5, \) and \( G_0(z) = -z^{-1}(1 + z)(5 - 4z + z^2)/5 \)
Case ii. \( b = 4, c = 1, \) and \( G_0(z) = -z^{-1}(1 + z)(1 - 4z + z^2) \)
Case iii. \( b = 4, c = 5, \) and \( G_0(z) = -z^{-1}(1 - z)(1 - 4z + 5z^2) \)

p. 281 Solution (6) (c): CHANGE \( \langle S^k \psi, \phi \rangle = -1/9 \) TO \( \langle S^k \psi, \phi \rangle = \pm 1/9 \)