Corrections to

Representations and Invariants of the Classical Groups

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p.25, l.10 REPLACE: We denote by s_0

BY: We denote by s_l

p.25, l.11 (display) REPLACE: s_0 BY: s_l

p.25, l.13 REPLACE:

$$J_+ = \begin{bmatrix} 0 & s_0 \\ s_0 & 0 \end{bmatrix}, \qquad J_+ = \begin{bmatrix} 0 & s_0 \\ -s_0 & 0 \end{bmatrix},$$

BY:

$$J_+ = \begin{bmatrix} 0 & s_l \\ s_l & 0 \end{bmatrix}, \qquad J_+ = \begin{bmatrix} 0 & s_l \\ -s_l & 0 \end{bmatrix},$$

p.25, l.-10 REPLACE: $s_0 a^t s_0$ BY: $s_l a^t s_l$

p.25, l.-7 REPLACE:

$$A = \begin{bmatrix} a & b \\ c & -s_0 a^t s_0 \end{bmatrix},$$

BY:

$$A = \begin{bmatrix} a & b \\ c & -s_l a^t s_l \end{bmatrix},$$

p. 25, l.-6 REPLACE: such that $b^t = -s_0 b s_0$ and $c^t = -s_0 c s_0$

BY: such that $b^t = -s_l b s_l$ and $c^t = -s_l c s_l$

p.25, l.-3 REPLACE:

$$A = \begin{bmatrix} a & b \\ c & -s_0 a^t s_0 \end{bmatrix},$$

BY:

$$A = \begin{bmatrix} a & b \\ c & -s_l a^t s_l \end{bmatrix},$$

p. 25, l.-2 REPLACE: such that $b^t = s_0 b s_0$ and $c^t = s_0 c s_0$

BY: such that $b^t = s_l b s_l$ and $c^t = s_l c s_l$

p.26, l.6 REPLACE:

$$S = \begin{bmatrix} 0 & 0 & s_0 \\ 0 & 1 & 0 \\ s_0 & 0 & 0 \end{bmatrix}.$$

BY:

$$S = \begin{bmatrix} 0 & 0 & s_l \\ 0 & 1 & 0 \\ s_l & 0 & 0 \end{bmatrix}.$$

p.26, l.12 REPLACE:

$$A = \begin{bmatrix} a & w & b \\ u & 0 & -w^t s_0 \\ c & -s_0 u^t & -s_0 a^t s_0 \end{bmatrix},$$

BY:

$$A = \begin{bmatrix} a & w & b \\ u & 0 & -w^t s_l \\ c & -s_l u^t & -s_l a^t s_l \end{bmatrix},$$

p.26, l.13 REPLACE: such that $b^t = -s_0 b s_0$ and $c^t = -s_0 c s_0$

BY: such that $b^t = -s_l b s_l$ and $c^t = -s_l c s_l$

p.30, l.14 REPLACE: $f_C(g)$ BY: $f_C^\pi(g)$

p.30, l.16 REPLACE: $f_{d\pi(A)C}(g)$ BY: $f^\pi_{d\pi(A)C}(g)$

p.31, l.-7 and -8 REPLACE:

$$X_{d\sigma(A)} f = X_A(f \circ \sigma) = X_{d\pi(A)}(f \circ \rho) = X_{d\rho(d\pi(A))} f.$$

Hence $d\sigma(A) = d\rho(d\pi(A))$.

BY:

$$X_{d\sigma(A)} f^\sigma = X_A(f \circ \sigma) = X_{d\pi(A)}(f \circ \rho) = X_{d\rho(d\pi(A))}(f \circ \rho).$$

Now evaluate at I to see that $d\sigma(A) = d\rho(d\pi(A))$.

p.32, l.7 REPLACE: $\{(A, -A) \,:\, A \in \mathfrak{g} \cap \mathfrak{h}\}$

BY: $\{(A, A) \,:\, A \in \mathfrak{g} \cap \mathfrak{h}\}$

p.32, l.24 REPLACE: Now assume $d\sigma(\mathfrak{g}) = \mathfrak{h}$.

BY: Now assume $d\sigma(\mathfrak{g}) = \mathfrak{h}$ and H is connected.

p.40, end of line 1 The last word should be HINT:

p.40, end of line 3 The last word should be "that"

p.45, l.-10 REPLACE: $\{g \in G \,:\, gg^t = I\}$

BY: $\{g \in \mathrm{GL}(n, \mathbb{C}) \,:\, gg^t = I\}$

p.49, l.-10 REPLACE:

Exercis (REMAINDER OF LINE DISAPPEARED IN TYPSETTING)

BY:

Exercise # 4 in Appendix A). However, in the case of algebraic groups, Theorem

p.94, l.6 REPLACE: Let $s_0 \in \mathrm{GL}(2l, \mathbb{C})$

BY: Let $s_l \in \mathrm{GL}(l, \mathbb{C})$

p.94, l.10 REPLACE:

$$\pi(\sigma) = \begin{bmatrix} s_\sigma & 0 \\ 0 & s_0 s_\sigma s_0 \end{bmatrix},$$

BY:

$$\pi(\sigma) = \begin{bmatrix} s_\sigma & 0 \\ 0 & s_l s_\sigma s_l \end{bmatrix},$$

p.95, l.8 REPLACE:

$$\phi(\sigma) = \begin{bmatrix} s_\sigma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s_0 s_\sigma s_0 \end{bmatrix},$$

BY:

$$\phi(\sigma) = \begin{bmatrix} s_\sigma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s_l s_\sigma s_l \end{bmatrix},$$

p.155, l.10 REPLACE: for all $x \in F$.

BY: for all $x \in F$. (HINT: Use Proposition 3.4.7.)

p.170, l.12 REPLACE: if $\phi \in \mathcal{J}$ then there exist

BY: if $\phi \in \mathcal{J}_+$ then there exist

p.174, l.−7 REPLACE: induction that $\mathcal{H} \cdot (\mathcal{P}\mathcal{J}_+)$ contains all polynomials

BY: induction that $\mathcal{H} \cdot (1 + \mathcal{P}\mathcal{J}_+)$ contains all polynomials

p.175, l.7 REPLACE: 4.1.4(1), which contradicts

BY: 4.1.4, which contradicts

p.183, l.−7 display REPLACE:

$$uZw = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{m-r,r} & O_{m-r} \end{bmatrix}$$

BY:

$$uZw = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{k-r,r} & O_{k-r,m-r} \end{bmatrix}$$

p.183, l.−5 display REPLACE:

$$X = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{m-r,r} & O_{k-r} \end{bmatrix}, \quad Y = \begin{bmatrix} I_r & O_{r,n-r} \\ O_{n-r,r} & O_{n-r} \end{bmatrix},$$

BY:

$$X = \begin{bmatrix} I_r & O_{r,n-r} \\ O_{k-r,r} & O_{k-r,n-r} \end{bmatrix}, \quad Y = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{n-r,r} & O_{n-r,m-r} \end{bmatrix},$$

p.184, l.−8 display REPLACE:

$$X = \begin{bmatrix} J_r & O_{r,k-r} \\ O_{k-r,r} & O_{n-r,k-r} \end{bmatrix} g.$$

BY:

$$X = \begin{bmatrix} J_r & O_{r,k-r} \\ O_{n-r,r} & O_{n-r,k-r} \end{bmatrix} g.$$

p.189, l.10 display REPLACE: $\prod_{j=1}^{k} y_j^{q_j}$ BY: $\prod_{j=1}^{m} y_j^{q_j}$

p.189, l.13 REPLACE: $z = (v_1, \ldots, v_k, v_1^*, \ldots, v_k^*)$

BY: $z = (v_1, \ldots, v_k, v_1^*, \ldots, v_m^*)$

p.198, l.-7 REPLACE: representation on $\mathbb{C}^n$ BY: representation on V

p.198, l.-5 REPLACE: space $\mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m)^{\mathrm{GL}(V)}$

BY: space $\mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m)^{\mathrm{GL}(V)}$

p.198, l.-3 REPLACE: acts on $\mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m)$

BY: acts on $\mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m)$

p.198, l.-1 **display** REPLACE: $\mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m)^{\mathrm{GL}(V)} = 0$

BY: $\mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m)^{\mathrm{GL}(V)} = 0$

p.199, l.2 **display** REPLACE: $\mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m)^{\mathrm{GL}(V)}$

BY: $\mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m)^{\mathrm{GL}(V)}$

p.199, l.4 REPLACE: complete contractions C_s BY: complete contractions λ_s

p.199, l.6 display REPLACE: C_s BY: λ_s

p.199, l.9 display REPLACE: C_s BY: λ_s

p.227, l.-1 REPLACE: general Capelli problem."

BY: general "Capelli problem."

p.254, l.-17 to l.-14 REPLACE:

Suppose for the sake . . . impossible. Hence $V_k = \mathcal{H}^k(\mathbb{C}^n)$.

BY:

We next prove (3). The intertwining property is easily checked (here it is understood that in $V^k \otimes \mathcal{H}^k(\mathbb{C}^n)$, $\mathfrak{g}'$ acts only on V^k and G acts only on $\mathcal{H}^k(\mathbb{C}^n)$). Suppose $\sum \psi_j(r^2) f_j = 0$. We may assume that $\{f_j\}$ is linearly independent and by homogeneity that $\psi_j(r^2) = c_j r^2$. This immediately implies $c_j = 0$, proving injectivity of the map. From (3) and Theorem 4.5.12 it follows that $\mathcal{H}^k(\mathbb{C}^n)$ is an irreducible G-module. Hence $V_k = \mathcal{H}^k(\mathbb{C}^n)$.

p.254, l.-7 REPLACE:

Statement (3) follows from (1) and (2) and Theorem 4.5.12. To prove (4) we only

BY:

To prove (4), we only

p.257, l.−3 REPLACE: *of size r such that* BY: *of size 2r such that*

p.258, l.18 REPLACE: it has degree $|\mu|$ BY: it has degree $|\mu|/2$

p.259, l.5 REPLACE: *such that* $|\mu| = r$ *and* BY: *such that* $|\mu| = 2r$ *and*

p.272, l.−1 REPLACE: $\epsilon(x^*)\epsilon(y^*) = -\epsilon(x^*)\epsilon(y^*)$ BY: $\epsilon(x^*)\epsilon(y^*) = -\epsilon(y^*)\epsilon(x^*)$

p.273, change displayed formula numbers:

(4.5.5) $\longrightarrow$ (6.5.5)

(4.5.6) $\longrightarrow$ (6.5.6)

(4.5.7) $\longrightarrow$ (6.5.7)

(4.5.8) $\longrightarrow$ (6.5.8)

p.275, change displayed formula number:

(4.5.9) $\longrightarrow$ (6.5.9)

p.276, change displayed formula numbers:

(4.5.10) $\longrightarrow$ (6.5.10)

(4.5.11) $\longrightarrow$ (6.5.11)

p.304, l.-9 REPLACE: π BY: π^λ

p.340, l.−16 REPLACE: $\gamma s_0 \gamma^t = I_{2l}$ BY: $\gamma s_{2l} \gamma^t = I_{2l}$

p.340, l−15 REPLACE: where s_0 is the matrix

BY: where s_{2l} is the matrix

p.340, l−14 REPLACE: corresponding to s_0 as in

BY: corresponding to s_{2l} as in

p.340, l−12 REPLACE:

$$\gamma g \gamma^{-1}(\gamma g \gamma^{-1})^t = \gamma g s_0 g^t \gamma^t = \gamma s_0 \gamma^t = I_{2l}.$$

BY:

$$\gamma g \gamma^{-1}(\gamma g \gamma^{-1})^t = \gamma g s_{2l} g^t \gamma^t = \gamma s_{2l} \gamma^t = I_{2l}.$$

p.340, l−9 REPLACE: defined by the equation $g^t g = I$.

BY: defined by the equation $g^t g = I_{2l}$.

p.342, l.−12 REPLACE: invariant volume forms on K/T, T, and K, respectively

BY: invariant volume forms on T, K/T, and K, respectively

p.371, l.6 REPLACE: (published by CAN, Amsterdam, 1992).

BY: (available at `http://young.sp2mi.univ-poitiers.fr/~marc/LiE`)

p.376, l.9 REPLACE: representation $(\sigma_k^\lambda, G^\lambda)$ BY: representation $(\sigma^\lambda, G^\lambda)$

p.377, l.-3 REPLACE: λ^* BY: λ^{T}

p.377, l.-2 REPLACE: $\lambda_1^* \geq \lambda_2^*$ BY: $\lambda_1^{\mathrm{T}} \geq \lambda_2^{\mathrm{T}}$

p.377, l.-1 REPLACE: λ_j^* BY: λ_j^{T}

p.378, l.1, l.2, and l.4 REPLACE: λ^* BY: λ^{T} (4 REPLACEMENTS)

p.384, l.10 REPLACE:

(1) The multiplicities $m_{\pi,\pi'}$ in (9.2.1) are either 0 or 1,

BY:

(1) The multiplicities $m_{\pi,\pi'}$ are either 0 or 1 and each $\pi \in \widehat{K}$ (resp. $\pi' \in \widehat{K'}$) occurs at most once in (9.2.1).

p.384, l.-3 REPLACE:

$G \times G'$ on Y is also multiplicity free.

BY:

$(\rho(G), \rho(G'))$ on Y is a dual reductive pair.

p.385, l.8 REPLACE: for $f \in \mathcal{P}(X)$ BY: for $f \in \mathcal{P}^d(X)$

p.385, l.-1 REPLACE:

This shows that $\mathcal{P}^d(X)$ is multiplicity-free for $G \times G'$

BY:

This shows that (G, G') acts as a dual pair on $\mathcal{P}^d(X)$.

p.386, l.5 REPLACE: Thus Y is multiplicity free for $K \times K'$.

BY: Thus (K, K') acts as a dual pair on Y.

p.386, l.10 and l.11 REPLACE:

As a $(G \times G')$-module, Y is also multiplicity free, since

BY:

We have (G, G') acting as a dual pair on Y, since

p.386, l.-1 REPLACE: $V_{(\tau)}$ BY: V_τ

p.394, l.7 REPLACE: λ^* BY: λ^{T}

p.394, l.9 REPLACE:

$e_1 \otimes e_2 \otimes \cdots \otimes e_{\lambda_1^*} \otimes e_1 \otimes e_2 \otimes \cdots \otimes e_{\lambda_2^*} \otimes \cdots \otimes e_1 \otimes e_2 \otimes \cdots \otimes e_{\lambda_q^*}$

BY:

$e_1 \otimes e_2 \otimes \cdots \otimes e_{\lambda_1^{\mathrm{T}}} \otimes e_1 \otimes e_2 \otimes \cdots \otimes e_{\lambda_2^{\mathrm{T}}} \otimes \cdots \otimes e_1 \otimes e_2 \otimes \cdots \otimes e_{\lambda_q^{\mathrm{T}}}$

p.394, l.10 REPLACE: λ_1^* BY: λ_1^{T}

p.394, l.11 REPLACE: $\lambda_1^* + 1, \ldots, \lambda_2^*$

BY: $\lambda_1^{\mathrm{T}} + 1, \ldots, \lambda_2^{\mathrm{T}}$

p.394, l.12 REPLACE: $\omega_{\lambda_1^*} \otimes \omega_{\lambda_2^*} \otimes \cdots \otimes \omega_{\lambda_q^*}$,

BY: $\omega_{\lambda_1^{\mathrm{T}}} \otimes \omega_{\lambda_2^{\mathrm{T}}} \otimes \cdots \otimes \omega_{\lambda_q^{\mathrm{T}}}$,

p.403, l.7 REPLACE: λ^* BY: λ^{T} (2 REPLACEMENTS)

p.403, l.8 REPLACE: $\mathrm{Col}(A^*)$ BY: $\mathrm{Col}(A^{\mathrm{T}})$

p.403, l.12 and l.13 REPLACE:

suppose $\lambda = \lambda^*$. Prove that $\sigma^\lambda(s) = 0$ for all odd permutations $s \in \mathfrak{S}_k$.

BY:

suppose $\lambda = \lambda^{\mathrm{T}}$. Prove that $\chi^\lambda(s) = 0$ for all odd permutations $s \in \mathfrak{S}_k$, where χ^λ is the character of σ^λ.

p.418, l.-3 REPLACE: $\mu_1 \geq \cdots \geq \mu_n$

BY: $\mu_1 \geq \cdots \geq \mu_n \geq 0$

p.434, l.11 REPLACE:

$$\mathcal{H}\mathcal{T}_r^{\otimes k} = \{u \in \mathcal{T}_r^{\otimes k} \,:\, u \cdot u = 0 \text{ for all } u \in \mathcal{B}_{k,r+1}(V, \omega)\}$$

BY:

$$\mathcal{H}\mathcal{T}_r^{\otimes k} = \{u \in \mathcal{T}_r^{\otimes k} \,:\, z \cdot u = 0 \text{ for all } z \in \mathcal{B}_{k,r+1}(V, \omega)\}$$

p.436, equation (10.3.4) REPLACE:

$$1 \leq m(r, \lambda) \leq \dim(G^\lambda)|\mathcal{M}(k, r)|$$

BY:

$$\dim(G^\lambda) \leq m(r, \lambda) \leq \dim(G^\lambda)|\mathcal{M}(k, r)|$$

p.436, l.-8 REPLACE: Let $r \geq 0$ BY: Let $r > 0$

p.487, l.−5 and l.−6 REPLACE:

and

$$\frac{d}{dt}(y^{-1}\theta(y)(I+t\theta(B))y(I+tB))|_{t=0} = \mathrm{Ad}(y^{-1})\theta(B)+B.$$

BY:

whereas the curve $t \mapsto y(I+tB)$ is tangent to Q at y provided

$$0 = \frac{d}{dt}(y^{-1}\theta(y)(I+t\theta(B))y(I+tB))|_{t=0} = \mathrm{Ad}(y^{-1})\theta(B)+B.$$

p.502, l.17 REPLACE:

algebraic groups by Theorem ??.

BY:

algebraic groups by Corollary 11.1.3.

p.502, l.−5 REPLACE:

$$(\exp yX_0)g(\exp -yX_0) = t\exp[(t^{-\alpha}-1)y+zX_0].$$

BY:

$$(\exp yX_0)g(\exp -yX_0) = t\exp[((t^{-\alpha}-1)y+z)X_0].$$

p.544, l.−4 REPLACE: $G = \mathrm{SO}(\mathbb{C}_n, B)$

BY: $G = \mathrm{SO}(\mathbb{C}^n, B)$

p.561, l.18 REPLACE: $\theta(g) = JgJ^{-1}$.

BY: $\theta = \theta_{n,n}$ as in Type AIII.

p.566, l.−15 REPLACE: set $\mathcal{J} = \{f_{\mathrm{top}} : f \in \mathcal{I}\}$.

BY: set $\mathcal{J} = \mathrm{span}\{f_{\mathrm{top}} : f \in \mathcal{I}\}$.

p.566, l.−11 REPLACE: It is even easier to prove that $\mathcal{J}$ is closed under addition.

BY: By definition $\mathcal{J}$ is closed under addition.

p.567, l.13 REPLACE: $\{f_{\mathrm{top}} : f \in \mathcal{I}_c\}$. BY: $\mathrm{span}\{f_{\mathrm{top}} : f \in \mathcal{I}_c\}$.

p.599, l.-7 REPLACE: Theorem 13.3.1 BY: Theorem A.3.1

p.600, l.5 REPLACE: /6, Theorem 14 BY: §6, Theorem 14

p.600, l.-2 REPLACE: Proposition 13.3.2 BY: Proposition A.3.2

p.601, l.-12 REPLACE: Theorem 13.3.3 BY: Theorem A.3.3

p.601, l.-5 REPLACE: Lemma 13.3.4 BY: Lemma A.3.4

p.603, l.-15 REPLACE: Corollary 13.3.5 BY: Corollary A.3.5

p.604, l.9 REPLACE: Proposition 13.3.6 BY: Proposition A.3.6

p.605, l.-3 REPLACE: Lemma 13.4.1 BY: Lemma A.4.1

p.607, l.-10 REPLACE: Lemma 13.4.2 BY: Lemma A.4.2

p.608, l.-12 REPLACE: Lemma 13.4.3 BY: Lemma A.4.3

p.609, l.17 REPLACE: Lemma 13.4.4 BY: Lemma A.4.4

p.609, l.21 REPLACE: Proposition 13.4.5 BY: Proposition A.4.5

p.609, l.-3 REPLACE: Corollary 13.4.6 BY: Corollary A.4.6

p.610, l.5 REPLACE: Theorem 13.4.7 BY: Theorem A.4.7

p.610, l.-16 REPLACE: Corollary 13.4.8 BY: Corollary A.4.8

p.611, l.11 REPLACE: Ch. I, / 5.2, Theorem 3 BY: Ch. I.5.2, Theorem 3

p.611, l.-7 REPLACE: Theorem 13.4.9 BY: Theorem A.4.9

p.611, l.-4 REPLACE: Corollary 13.4.10 BY: Corollary A.4.10

General Typesetting Error: The ligature "fi" (as in the words: define, finite, field, fixed, satisfied, first, suffices, find) was omitted on the following pages:

15-32, 226, 227, 273-275, 278, 279, 598-611