Corrections to

Representations and Invariants of the Classical Groups
by Roe Goodman and Nolan R. Wallach
(1998 hard-cover edition)

Revised January 18, 2002

Note: Most of the following corrections are incorporated into the 1999 (paperback) printing.

p.15, l.−14 to l.−8 (proof of assertion (2)) REPLACE:

We may assume that . . . this proves (2).

BY:

The point evaluations \( \{\delta_x\}_{x \in X} \) span \( V^* \). Choose \( x_i \in X \) so that \( \{\delta_{x_1}, \ldots, \delta_{x_q}\} \) is a basis for \( V^* \) and let \( \{g_1, \ldots, g_q\} \) be the dual basis for \( V \). Then we can write

\[
R(x)g_j = \sum_{i=1}^q c_{ij}(x) g_i
\]

for \( x \in X \). Since

\[
c_{ij}(x) = \langle R(x)g_j, \delta_{x_i} \rangle = g_j(xix),
\]

we see that \( x \mapsto c_{ij}(x) \) is a regular function on \( X \). This proves (2).

p.15, l.−3 REPLACE: \( \{f_1, \ldots, f_m\} \subset \rho^*\text{Aff}(G) \) BY: \( \{f_1, \ldots, f_n\} \subset \Phi^*\text{Aff}(G) \)

p.16, l.1 to l.26 REPLACE :

The following theorem shows that . . .

(statement and proof of Theorem 1.1.14)

. . . so \( \sigma^{-1} \) is regular (see Section A.4.3). □

BY:

Example

Let \( B \) be a bilinear form on \( \mathbb{C}^n \). We define a multiplication \( \ast_B \) on \( \mathbb{C}^{n+1} \) by

\[
\begin{bmatrix} x \\ \lambda \end{bmatrix} \ast_B \begin{bmatrix} y \\ \mu \end{bmatrix} = \begin{bmatrix} x + y \\ \lambda + \mu + B(x, y) \end{bmatrix}
\]

for \( x, y \in \mathbb{C}^n \) and \( \lambda, \mu \in \mathbb{C} \). From the bilinearity of \( B \) we calculate easily that this multiplication is associative. Since

\[
\begin{bmatrix} x \\ \lambda \end{bmatrix} \ast_B \begin{bmatrix} -x \\ -\lambda + B(x, x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
\]

we have
we conclude that $*_B$ defines a group structure on $\mathbb{C}^{n+1}$ with 0 as the identity element. Multiplication and inversion are regular maps, so by Theorem 1.1.13 there is a linear algebraic group $G_B$ with $\text{Aff}(G_B) \cong \text{Aff}(\mathbb{C}^{n+1})$ as a $\mathbb{C}$-algebra and $G_B \cong (\mathbb{C}^{n+1},*_B)$ as a group.

We can use the proof of Theorem 1.1.13 to obtain an explicit matrix realization of $G_B$. Let $f_i(x) = x_i$ for $x \in \mathbb{C}^{n+1}$ and let $g_i \in (\mathbb{C}^{n})^\ast$ for $i = 1, \ldots, n$ be the linear functionals such that

$$B(x, y) = \sum_{i=1}^{n} f_i(x) g_i(y)$$

for $x, y \in \mathbb{C}^n$. Let $f_0(x) = 1$ for all $x \in \mathbb{C}^{n+1}$. For $f \in \text{Aff}(\mathbb{C}^{n+1})$ and $y \in \mathbb{C}^{n+1}$ let $R(y) f(x) = f(x*_By)$. From the definition of the multiplication $*_B$ we have $R(y) f_0 = f_0$, $R(y) f_i = f_i + f_i(y)$ for $1 \leq i \leq n$, and

$$R(y) f_{n+1} = f_{n+1} + f_{n+1}(y) + \sum_{i=1}^{n} g_i(y) f_i$$

(we define $g_i(y) = g_i(\bar{y})$, where $\bar{y}$ is the projection of $y$ onto $\mathbb{C}^n$). Thus the $(n+2)$-dimensional subspace $V$ of $\text{Aff}(\mathbb{C}^{n+1})$ spanned by the functions $f_0, \ldots, f_{n+1}$ is invariant under $R(y)$. Let $\Phi(y)$ be the restriction of $R(y)$ to $V$. Then $\Phi(y)$ has the matrix

$$\begin{bmatrix}
1 & f_1(y) & \cdots & f_n(y) & f_{n+1}(y) \\
0 & 1 & \cdots & 0 & g_1(y) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & g_n(y) \\
0 & 0 & \cdots & 0 & 1
\end{bmatrix}$$

relative to the ordered basis $\{f_0, f_1, \ldots, f_{n+1}\}$ for $V$. Since $f_i$ and $g_i$ are linear functions and $\{f_i(y)\}$ are the coordinates of $y$, it is clear that $G_B = \Phi(\mathbb{C}^{n+1})$ is a closed subgroup of $\text{GL}(n+2, \mathbb{C})$ that is isomorphic to $(\mathbb{C}^{n+1},*_B)$ as a group and as an affine algebraic set.

p.25, l.10 REPLACE: We denote by $s_0$

BY: We denote by $s_l$

p.25, l.11 (display) REPLACE: $s_0$ BY: $s_l$

p.25, l.13 REPLACE:

$$J_+ = \begin{bmatrix} 0 & s_0 \\ s_0 & 0 \end{bmatrix}, \quad J_+ = \begin{bmatrix} 0 & s_0 \\ -s_0 & 0 \end{bmatrix},$$

BY:

$$J_+ = \begin{bmatrix} 0 & s_l \\ s_l & 0 \end{bmatrix}, \quad J_+ = \begin{bmatrix} 0 & s_l \\ -s_l & 0 \end{bmatrix},$$

p.25, l.10 REPLACE: $s_0 a^t s_0$ BY: $s_l a^t s_l$
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p.25, l.-7 REPLACE:

\[ A = \begin{bmatrix} a & b \\ c & -s_0a's_0 \end{bmatrix}, \]

BY:

\[ A = \begin{bmatrix} a & b \\ c & -s_1a's_1 \end{bmatrix}, \]

p. 25, l.-6 REPLACE: such that \( b^t = -s_0b's_0 \) and \( c^t = -s_0c's_0 \)

BY: such that \( b^t = -s_1b's_1 \) and \( c^t = -s_1c's_1 \)

p.25, l.-3 REPLACE:

\[ A = \begin{bmatrix} a & b \\ c & -s_0a's_0 \end{bmatrix}, \]

BY:

\[ A = \begin{bmatrix} a & b \\ c & -s_1a's_1 \end{bmatrix}, \]

p. 25, l.-2 REPLACE: such that \( b^t = s_0b's_0 \) and \( c^t = s_0c's_0 \)

BY: such that \( b^t = s_1b's_1 \) and \( c^t = s_1c's_1 \)

p.26, l.6 REPLACE:

\[ S = \begin{bmatrix} 0 & 0 & s_0 \\ 0 & 1 & 0 \\ s_0 & 0 & 0 \end{bmatrix}. \]

BY:

\[ S = \begin{bmatrix} 0 & 0 & s_1 \\ 0 & 1 & 0 \\ s_1 & 0 & 0 \end{bmatrix}. \]

p.26, l.12 REPLACE:

\[ A = \begin{bmatrix} a & w & b \\ u & 0 & -w's_0 \\ c & -s_0u' & -s_0a's_0 \end{bmatrix}, \]

BY:

\[ A = \begin{bmatrix} a & w & b \\ u & 0 & -w's_1 \\ c & -s_1u' & -s_1a's_1 \end{bmatrix}, \]

p.26, l.13 REPLACE: such that \( b^t = -s_0b's_0 \) and \( c^t = -s_0c's_0 \)

BY: such that \( b^t = -s_1b's_1 \) and \( c^t = -s_1c's_1 \)

p.31, l.–8 to l.–1 REPLACE PRINTED TEXT BY:

\( X_AI_G \subset I_G \). Write \( \sigma = \pi|_G \) and take \( f = f_C \circ \pi \) for \( C \in \text{End}(V) \). Then \( X_A(f_C \circ \sigma)(I) = X_A(f_C \circ \pi)(I) \), and hence \( d\sigma(A) = d\pi(A) \) by (1.2.9).
(3): Write $g = \text{Lie}(G)$ and $\mathfrak{h} = \text{Lie}(H)$. By (1), $\text{Lie}(G \cap H) \subset g \cap \mathfrak{h}$. Let $X = G \times H$ and define $\varphi : X \to \text{GL}(n, \mathbb{C})$ by $\varphi(g, h) = gh^{-1}$. Set $Y = \varphi(X)$ and $F_y = \varphi^{-1}\{y\}$. Then $F_{gh^{-1}} = \{(gz, hz) : z \in G \cap H\}$, and hence $\dim F_{gh^{-1}} = \dim(G \cap H)$ for all $(g, h) \in X$. Since $\text{Ker } d\varphi(1, 1) = \{(A, -A) : A \in g \cap \mathfrak{h}\}$ and $d\varphi(1, 1) = dL_g dR_h d\varphi(1, 1)$, we have $\dim \text{Ker } d\varphi(g, h) = \dim(g \cap \mathfrak{h})$ for all $(g, h) \in X$. Proposition A.3.6 now implies that $\dim(G \cap H) = \dim(g \cap \mathfrak{h})$, hence $\text{Lie}(G \cap H) = g \cap \mathfrak{h}$.

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p.30, l.4 REPLACE: Corollary A.3.6 BY: Corollary A.3.5

p.32, l.2 REPLACE: $=[\text{Ad}(g)A, \text{Ad}(g)A]$, BY: $=[\text{Ad}(g)A, \text{Ad}(g)B]$,

p.39, l.15 REPLACE: $g_u$, BY: $g_u$

p.39, l.13 REPLACE:
subset of End($V$) and $G_u$ is an algebraic subset of GL($V$).

BY:
subset of $M_n(\mathbb{C})$ and $G_u$ is an algebraic subset of GL($n, \mathbb{C}$).

p.39, l.4 and l.3 REPLACE:
Decompose $\mathbb{C}^n$ into spaces $W_\lambda = \{w \in \mathbb{C}^n : (H - \lambda I)^p w = 0 \text{ for some } p\}$. Show that $XW_\lambda \subset W_{\lambda+2}$.)

BY:
Show that $[H, X^k] = 2kX^k$. Then consider the eigenvalues of ad$H$ on $M_n(\mathbb{C})$.

p.44, l.9 REPLACE:
Hence $\rho^{-1}$ is regular by Theorem 1.1.14.

BY:
Clearly $\rho^*(\text{Aff}(H)) = \text{Aff}(G)$, so $\rho^{-1}$ is regular.

p.49, l.4 (Exercise #1) REPLACE:
1. Check the assertion in (1.4.2) above.

BY:
1. Define a real form Sp($p, q$) of Sp($p+q, \mathbb{C}$) analogous to the real form U($p, q$) of GL($p+q, \mathbb{C}$).

p.49, l.7 and l.8 (Exercise #3) REPLACE:
Let $\psi \in \text{End}(\mathbb{C}^{2n})$ act by

$$\psi[z_1, \ldots, z_n, z_{n+1}, \ldots, z_{2n}] = [\bar{z}_{n+1}, \ldots, \bar{z}_{2n}, -\bar{z}_1, \ldots, -\bar{z}_n]$$

BY:
Let $\psi$ be the real linear transformation of $\mathbb{C}^{2n}$ defined by

$$\psi[z_1, \ldots, z_n, z_{n+1}, \ldots, z_{2n}] = [\bar{z}_{n+1}, \ldots, \bar{z}_{2n}, -\bar{z}_1, \ldots, -\bar{z}_n]$$
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p.51, formula (2.1.1) REPLACE: \[ \prod_{k=1}^{n} \] BY: \[ \prod_{k=1}^{l} \]

p.66, l.-7 REPLACE:
\[ \sigma_k(g)f(x) = (-cx + d)^k f \left( \frac{ax - b}{-cx + d} \right). \]
BY:
\[ \sigma_k(g)f(x) = (cx + a)^k f \left( \frac{dx + b}{cx + a} \right). \]

p.68, l.10 REPLACE: \[ P(G) = \text{Span}\{d\theta : \theta \in \mathcal{X}(H)\} \] BY: \[ P(G) = \{d\theta : \theta \in \mathcal{X}(H)\} \]

p.77, Figure 2.2 REPLACE: \[ \varepsilon_l - \varepsilon_{l+1} \] BY: \[ \varepsilon_{l-1} - \varepsilon_l \]

p.77, l.-13 and -12 REPLACE:
as in Type A,
BY:
and \( \varepsilon_i + \varepsilon_l = \alpha_i + \cdots + \alpha_l \),

p.78, Figure 2.3 REPLACE: \[ \varepsilon_l - \varepsilon_{l+1} \] BY: \[ \varepsilon_{l-1} - \varepsilon_l \]

p.82, l.-17 REPLACE:
\[ \alpha_i + \cdots + \alpha_j \quad \text{for } 1 \leq i < j < l \]
BY:
\[ \alpha_i + \cdots + \alpha_j \quad \text{for } 1 \leq i < j \leq l \]

p.94, l.6 REPLACE: Let \( s_0 \in \text{GL}(2l, \mathbb{C}) \)
BY: Let \( s_l \in \text{GL}(l, \mathbb{C}) \)

p.94, l.10 REPLACE:
\[ \pi(\sigma) = \begin{bmatrix} s_\sigma & 0 \\ 0 & s_0 s_\sigma s_0 \end{bmatrix}, \]
BY:
\[ \pi(\sigma) = \begin{bmatrix} s_\sigma & 0 \\ 0 & s_l s_\sigma s_l \end{bmatrix}, \]

p.95, l.8 REPLACE:
\[ \phi(\sigma) = \begin{bmatrix} s_\sigma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s_0 s_\sigma s_0 \end{bmatrix}, \]
BY:
\[ \phi(\sigma) = \begin{bmatrix} s_\sigma & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s_l s_\sigma s_l \end{bmatrix}. \]
p.95, l.−14 REPLACE: $O(2l + 1, \mathbb{C})$ BY: $O(B, \mathbb{C})$

p.169, l.−14 REPLACE:
From Theorem 3.3.6 we have a

BY:
From Proposition 3.1.6 we have the

p.170, l.12 REPLACE: if $\phi \in J$ then there exist

BY: if $\phi \in J_+$ then there exist

p.172, l.7 REPLACE:

\[
\sigma_I - x_1 = \sigma_1 - x_1 = x_2 + \cdots + x_n
\]

BY:

\[
\sigma_I - x'_1 = \sigma_1 - x_1 = x_2 + \cdots + x_n
\]

p.172, l.−14 REPLACE: $f(x) - a_I \sigma^I$ BY: $f(x) - a \sigma^I$

p.174, l.−7 REPLACE: induction that $\mathcal{H} \cdot (\mathcal{P} J_+)$ contains all polynomials

BY: induction that $\mathcal{H} \cdot (1 + \mathcal{P} J_+)$ contains all polynomials

p.175, l.7 REPLACE:

4.1.4(1), which contradicts

BY:

4.1.4, which contradicts

p.176, l.2 REPLACE: $g = 0. \square$ BY: $g = 0.$

p.180, l.14 REPLACE: $\rho(g^{-1})v_n$ BY: $\rho(g^{-1})v_m$

p.181, l.2 REPLACE: $f(x \rho(g^{-1}), \rho(g)y), \quad x \in X, \quad y \in Y.$ BY: $f(x \rho(g^{-1}), \rho(g)y)$

p.181, l.7 REPLACE: for $g \in G$ and $x \in X, y \in Y.$ BY: for $g \in \text{GL}(V)$

p.181, l.−15 REPLACE: $i = 1, \ldots, m, \quad j = 1, \ldots, k$ BY: $i = 1, \ldots, k, \quad j = 1, \ldots, m$

p.182, l.−5 REPLACE: $i \neq j$ BY: $i < j$

p.183, l.−7 DISPLAY REPLACE:

\[
uZW = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{m-r,r} & O_{m-r} \end{bmatrix}
\]

BY:

\[
uZW = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{k-r,r} & O_{k-r,m-r} \end{bmatrix}
\]
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p.183, l.−5 

\[ X = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{m-r,r} & O_{k-r} \end{bmatrix}, \quad Y = \begin{bmatrix} I_r & O_{r,n-r} \\ O_{n-r,r} & O_{n-r} \end{bmatrix}, \]

BY:

\[ X = \begin{bmatrix} I_r & O_{r,n-r} \\ O_{k-r,r} & O_{k-r,n-r} \end{bmatrix}, \quad Y = \begin{bmatrix} I_r & O_{r,m-r} \\ O_{n-r,r} & O_{n-r,m-r} \end{bmatrix}, \]

p.184, l.−8 

\[ X = \begin{bmatrix} J_r & O_{r,k-r} \\ O_{k-r,r} & O_{n-r,k-r} \end{bmatrix} g. \]

BY:

\[ X = \begin{bmatrix} J_r & O_{r,k-r} \\ O_{n-r,r} & O_{n-r,k-r} \end{bmatrix} g. \]

p.184, l.−3 

(SFT, Free Case) BY: (SFT, Free Case) Let \( V = \mathbb{C}^n \).

p.184, l.−2 

dim \( V \geq \min(k, m) \) BY: \( n \geq \min(k, m) \)

p.185, l.6, l.7, l.10 

(SFT) \( V^n \)

p.189, l.10 

\[ \prod_{j=1}^{k} y_j^{q_j} \]

BY:

\[ \prod_{j=1}^{m} y_j^{q_j} \]

p.189, l.13 

\( z = (v_1, \ldots, v_k, v'_1, \ldots, v'_m) \) 

BY: \( z = (v_1, \ldots, v_k, v'_1, \ldots, v'_m) \)

p.198, l.−7 

representation on \( \mathbb{C}^n \) 

BY: representation on \( V \)

p.198, l.−5 

space \( \mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m)_{\text{GL}(V)} \) 

BY: space \( \mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m)_{\text{GL}(V)} \)

p.198, l.−3 

acts on \( \mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m) \) 

BY: acts on \( \mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m) \)

p.198, l.−1 

\[ \mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m)_{\text{GL}(V)} = 0 \] 

BY: \[ \mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m)_{\text{GL}(V)} = 0 \]

p.199, l.2 

\[ \mathcal{P}^{[p,q]}(V^k \otimes (V^*)^m)_{\text{GL}(V)} \] 

BY: \[ \mathcal{P}^{[p,q]}(V^k \oplus (V^*)^m)_{\text{GL}(V)} \]
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p.199, l.4 REPLACE: complete contractions \(C_s\)
BY: complete contractions \(\lambda_s\)

p.199, l.6 display REPLACE: \(C_s\)
BY: \(\lambda_s\)

p.199, l.9 display REPLACE: \(C_s\)
BY: \(\lambda_s\)

p.211, l.7 REPLACE: \(EM = M\) BY: \(EM \subset M\)

p.211, l.−5 REPLACE: \(\text{Span}\{\rho(G)u\} = Z_{\lambda}\) BY: \(\text{Span}\{\rho(G)f\} = Z_{\lambda}\)

p.211, l.−1 REPLACE: \(u \in \mathcal{R}^G\) BY: \(r \in \mathcal{R}^G\)

p.218, l.−10 REPLACE:
\[(V^k)^*\] dual to the coordinates \(x_{ij}\) on \(V^k\).
BY:
\(V^*\) dual to the coordinates \(x_{ij}\) on \(V\).

p.219, l.13 REPLACE: \(\rho(g)D_{ij}\rho(g^{-1})\) BY: \(\rho(g)\Delta_{ij}\rho(g^{-1})\)

p.224, l.9 REPLACE: \(\xi^* \in V^*\) BY: \(\xi \in V^*\)

p.226 between l.5 and l.6 INSERT:

4.5.8 Exercises

1. Let \(G = \text{GL}(n, \mathbb{C})\) and \(V = M_{n,p}(\mathbb{C}) \oplus M_{n,q}(\mathbb{C})\). Let \(g \in G\) act on \(V\) by \(g \cdot (x \oplus y) = gx \oplus (g^t)^{-1}y\) for \(x \in M_{n,p}(\mathbb{C})\) and \(y \in M_{n,q}(\mathbb{C})\). Note that the columns \(x_i\) of \(x\) transform as vectors in \(\mathbb{C}^n\) and the columns \(y_j\) of \(y\) transform as covectors in \((\mathbb{C}^n)^*\).
(a) Let \(p_−\) be the subspace of \(\mathbb{D}(V)\) spanned by the operators of multiplication by \((x_i)^t \cdot y_j\) for \(1 \leq i \leq p\), \(1 \leq j \leq q\). Let \(p_+\) be the subspace of \(\mathbb{D}(V)\) spanned by the operators \(\Delta_{ij} = \sum_{r=1}^n \frac{\partial}{\partial x_r} \frac{\partial}{\partial y_r}\) for \(1 \leq i \leq p\), \(1 \leq j \leq q\). Prove that \(p_\pm \subset \mathbb{D}(V)^G\).
(b) Let \(\mathfrak{t}\) be the subspace of \(\mathbb{D}(V)\) spanned by the operators \(E_{ij}^{(x)} + \frac{k}{2} \delta_{ij}\) (with \(1 \leq i, j \leq p\)) and \(E_{ij}^{(y)} + \frac{k}{2} \delta_{ij}\) (with \(1 \leq i, j \leq q\)), where \(E_{ij}^{(x)}\) is defined by equation (4.5.27) and \(E_{ij}^{(y)}\) is similarly defined with \(x_{ij}\) replaced by \(y_{ij}\). Prove that \(\mathfrak{t} \subset \mathbb{D}(V)^G\).
(c) Prove the commutation relations \([\mathfrak{t}, \mathfrak{t}] \subset \mathfrak{t}\), \([\mathfrak{t}, p_\pm] = p_\pm\), \([p_-, p_+] \subset \mathfrak{t}\). (d) Set \(\mathfrak{g}' = p_+ + \mathfrak{t} + p_-\). Prove that \(\mathfrak{g}'\) is isomorphic to \(\mathfrak{gl}(p + q, \mathbb{C})\), and that \(\mathfrak{t} \cong \mathfrak{gl}(p, \mathbb{C}) \oplus \mathfrak{gl}(q, \mathbb{C})\).
(e) Prove that \(\mathbb{D}(V)^G\) is generated by \(\mathfrak{g}'\). (HINT: Use Theorems 4.2.1 and 4.5.16. Note that there are four possibilities for contractions to obtain \(G\)-invariant polynomials on \(V \oplus V^*\): (1) vector and covector in \(V\); (2) vector and covector in \(V^*\); (3) vector from . . .
V and covector from $V^*$; (4) covector from $V$ and vector from $V^*$. Show that the contractions of types (1) and (2) furnish symbols for bases of $p_{\pm}$, and that contractions of type (3) and (4) furnish symbols for a basis of $\mathfrak{t}$.

**p.226, l.7** REPLACE:

The finiteness result in Theorem 4.1.1, due to Hilbert, was a major

BY:

Theorem 4.1.1 (the proof given is due to Hurwitz) was a major

**p.227, l.-1** REPLACE:

general Capelli problem.”

BY: general “Capelli problem.”

**p.237, l.10** REPLACE:

$\rho(p) = 0, 1, \ldots, [k/2]$. (where $\varpi_0 = 0$.)

BY:

$\rho(p) = 0, 1, \ldots, [k/2]$ (where $\varpi_0 = 0$.)

**p.237, l.12** REPLACE:

$l - p \bigoplus_{k=0}^{i-p}$

BY:

$l - p \bigoplus_{k=0}^{i-p}$

**p.243, l.2** REPLACE:

If we choose $-\Phi_+$

BY:

If we choose $-\Phi^+$

**p.249, l.9** REPLACE:

$z^{m_1 + \cdots + m_n}$

BY:

$z^{m_1 + \cdots + m_n}$

**p.250, l.8** REPLACE:

$O(n, \mathbb{C})$

BY:

$O(B, \mathbb{C})$

**p.254, l.13** REPLACE:

We can choose $g_1 \in G$ so that $G = G^0 \cup g_1 G^0$ and $\rho(g_1) \phi^k = \pm \phi^k$

BY:

We can choose $g_0 \in G$ so that $G = G^0 \cup g_0 G^0$ and $\rho(g_0) \phi^k = \phi^k$

**p.255, l.8** REPLACE:

$\sum \mu_i$

BY:

$\sum i\mu_i$

**p.256, l.9** REPLACE:

$\text{depth}(\mu) \leq r$

BY:

$\text{depth}(\nu) \leq r$

**p.257, l.3** REPLACE:

of size $r$ such that

BY:

of size $2r$ such that

**p.258, l.18** REPLACE:

it has degree $|\mu|$,

BY:

it has degree $|\mu|/2$

**p.259, l.5** REPLACE:

such that $|\mu| = r$ and

BY:

such that $|\mu| = 2r$ and

**p.270, l.3** REPLACE:

$2\gamma(v_i)^2 = \beta(v_i, v_i)$

BY:

$\{\gamma(v_i), \gamma(v_j)\} = \beta(v_i, v_j)$

**p.272, l.1** REPLACE:

$\epsilon(x^*)\epsilon(y^*) = -\epsilon(x^*)\epsilon(y^*)$

BY:

$\epsilon(x^*)\epsilon(y^*) = -\epsilon(y^*)\epsilon(x^*)$

**p.273, l.12** REPLACE:

We combine them into a linear map

BY:

When dim $V$ is even, we combine these operators to obtain a linear map
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**p.274, l.1** REPLACE:

Let \( \{ e_1, \ldots, e_k \} \) be a basis for \( W \), where \( k = n/2 \), and let \( \{ e_{-1}, \ldots, e_{-k} \} \) be the basis BY:

Let \( \{ e_1, \ldots, e_l \} \) be a basis for \( W \), where \( l = n/2 \), and let \( \{ e_{-1}, \ldots, e_{-l} \} \) be the basis

**p.274, l.3** REPLACE: with \( 1 \leq j_1 < \cdots < j_p \leq k \) BY: with \( 1 \leq j_1 < \cdots < j_p \leq l \)

**p.275, l.2, l.3, l.4** REPLACE:

Since the range of \( T \) is spanned by \( 2^l \) vectors and \( \dim(\wedge^W) = 2^l \), we conclude that \( T \) is bijective.

BY:

We will prove that \( T\gamma(w + w^*) = \gamma'(w + w^*)T \) for \( w \in W \) and \( w^* \in W^* \). This will imply that \( \text{Ker}T = 0 \), since \( \gamma(W + W^*) \) acts irreducibly, and hence that \( \dim Z = 1 \).

**p.276, l.−1** REPLACE: \((1)^r e_{j_1} \wedge \cdots \) BY: \((-1)^r e_{j_1} \wedge \cdots \)

**p.277, l.7** REPLACE: \( \dim V = 2^l + 1 \) is odd, BY: \( \dim V = 2^l + 1 \) is odd with \( l \geq 1 \),

**p.277, l.−9** REPLACE: We use the tensor-product model

BY:

Let \( l \geq 1 \) (the case \( \dim V = 1 \) is left to the reader) and use the model

**p.278, l.−7** REPLACE: dimension \( 2^{\dim V} \), BY: dimension \( 2^{\dim V_0} \),

**p.279, l.12** REPLACE: \( (x_1 e_1 + \cdots + x_n e_n)^2 \) BY: \( 2(x_1 e_1 + \cdots + x_n e_n)^2 \)

**p.279, l.16** REPLACE: \( \sum_{i=1}^{n} \) BY: \( \frac{1}{2} \sum_{i=1}^{n} \)

**p.279, l.−13** REPLACE: \( (2 \sum R_{ijji})I \) BY: \( (1/2) \sum R_{ijji} \)

**p.279, l.−11** REPLACE: algbera BY: algebra

**p.281, l.6** REPLACE: \([\phi(X), \lambda(v)] \) BY: \([\phi(X), \gamma(v)] \)

**p.281, l.−1** REPLACE: spin representation BY: space of spinors

**p.284, l.−15** REPLACE: dominant weight BY: highest weight

**p.285, l.−12** REPLACE: \( c : V \rightarrow \text{Cliff}(V, \beta) \) BY: \( \gamma : V \rightarrow \text{Cliff}(V, \beta) \)

**p.286, l.4** REPLACE: \( \rho(x_1) = 0 \) BY: \( \tilde{\gamma}(x_1) = 0 \)

**p.286, l.5** REPLACE: \( \rho(x_1) \) BY: \( \tilde{\gamma}(x_1) \)

**p.286, l.8** REPLACE:

\( \rho_{\pm}(x_1) = \pm \mu I \) for some \( \mu \in \mathbb{C} \).

BY:

\( \tilde{\gamma}_{\pm}(x_1) = \mu_{\pm} I \) for some \( \mu_{\pm} \in \mathbb{C} \).
\textbf{p.286, l.9} REPLACE:
\[ \rho_\pm(e_0) \text{ is invertible, so } \mu = 0. \]
BY:
\[ \tilde{\gamma}_\pm(e_0) \text{ is invertible, so } \mu_\pm = 0. \]

\textbf{p.286, l.18} REPLACE:
\[ \text{Hence } O(V, \beta) \text{ is generated by reflections.} \]
BY:
\[ (3) \text{ O}(V, \beta) \text{ is generated by reflections.} \]

\textbf{p.286, l.−5} REPLACE:
\[ \text{a product of reflections.} \]
BY:
\[ \text{a product of reflections, proving (3).} \]

\textbf{p.288, l.−15 and l.−14} REPLACE:
\[ \text{These subalgebras are spanned by elements of the form } R_{x,y} \text{ where } x, y \in V \text{ satisfy} \]
BY:
\[ \text{By Lemma 6.2.1 these subalgebras are spanned by elements } R_{x,y} \text{ where } x, y \in V \text{ satisfy} \]

\textbf{p.288, l.−5} REPLACE:
\[ = \frac{1}{2} \beta(y, y) \beta(x, z) \gamma(x), \]
BY:
\[ = \frac{1}{2} \beta(y, y) \beta(x, z) \gamma(x) = 0, \]

\textbf{p.288, l.−3} REPLACE:
\[ u(t) \gamma(z) u(-t) = \gamma(z) + t[\gamma(x) \gamma(y), \gamma(z)] + \frac{t^2}{2} \beta(y, y) \beta(x, z) \gamma(x) \]
BY:
\[ u(t) \gamma(z) u(-t) = \gamma(z) + t[\gamma(x) \gamma(y), \gamma(z)] \]

\textbf{p.288, l.−2} REPLACE:
\[ = \gamma(z) + t \gamma(R_{x,y} z) + \frac{t^2}{2} \beta(y, y) \beta(x, z) \gamma(x) \]
BY:
\[ = \gamma(z) + t \gamma(R_{x,y} z) \]

\textbf{p.294, l.−6} REPLACE:
\[ (g) \text{ Spin}(5, 1)^0 \cong SU(1, 3). \]
BY:
\[ (g) \text{ Spin}(5, 1)^0 \cong SU^*(4) \cong SL(2, \mathbb{H}) \text{ (see 1.4.6, Exercise # 3).} \]
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p.333, l.3 replace: \( Q \in \Phi^+ \) by: \( Q \subset \Phi^+ \)

p.336, l. -7 replace:
for every \( Q \subset \Phi^+ \) and has multiplicity one.
by:
for every \( Q \subset \Phi^+ \).

p.340, l. -16 replace:
\[
\gamma s_0 \gamma^t = I_{2l}
\]
by:
\[
\gamma s_{2l} \gamma^t = I_{2l}
\]

p.340, l. -15 replace: where \( s_0 \) is the matrix
by: where \( s_{2l} \) is the matrix

p.340, l. -14 replace: corresponding to \( s_0 \) as in
by: corresponding to \( s_{2l} \) as in

p.340, l. -12 replace:
\[
\gamma g \gamma^{-1} (\gamma g \gamma^{-1})^t = \gamma g s_0 g^t \gamma^t = \gamma s_0 \gamma^t = I_{2l}.
\]
by:
\[
\gamma g \gamma^{-1} (\gamma g \gamma^{-1})^t = \gamma g s_{2l} g^t \gamma^t = \gamma s_{2l} \gamma^t = I_{2l}.
\]

p.340, l. -9 replace: defined by the equation \( g^t g = I \).
by: defined by the equation \( g^t g = I_{2l} \).

p.354, l. -1 replace: irreducible \( g \)-module by: irreducible \( \mathfrak{h} \)-module

p.434, l.11 replace:
\[
\mathcal{H}T^{\otimes k}_r = \{ u \in T^{\otimes k}_r : u \cdot u = 0 \text{ for all } u \in B_{k,r+1}(V,\omega) \}
\]
by:
\[
\mathcal{H}T^{\otimes k}_r = \{ u \in T^{\otimes k}_r : z \cdot u = 0 \text{ for all } z \in B_{k,r+1}(V,\omega) \}
\]

p.436, equation (10.3.4) replace:
\[
1 \leq m(r,\lambda) \leq \dim(G^\lambda) |\mathcal{M}(k,r)|
\]
by:
\[
\dim(G^\lambda) \leq m(r,\lambda) \leq \dim(G^\lambda) |\mathcal{M}(k,r)|
\]

p.436, l. -8 replace: Let \( r \geq 0 \) by: Let \( r > 0 \)

p.467, l. -3 replace: \( \text{Aff}(G/N) \) by: \( \pi^* \text{Aff}(G/N) \)
p.467, l.\(-1\) REPLACE: translates if \(f\) BY: translates of \(f\)

p.485, l.\(-4\) REPLACE: \(X_A\) BY: \(X_G\)

p.486, l.\(-4\) REPLACE: \(V_i \subset V_{i-1}\) BY: \(V_i^0 \subset V_{i-1}^0\)

p.487, l.\(-5\) and l.\(-6\) REPLACE:

\[
\frac{d}{dt}(y^{-1}\theta(y)(I + t\theta(B))y(I + tB))|_{t=0} = \text{Ad}(y^{-1})\theta(B) + B.
\]

BY:

whereas the curve \(t \mapsto y(I + tB)\) is tangent to \(Q\) at \(y\) provided

\[
0 = \frac{d}{dt}(y^{-1}\theta(y)(I + t\theta(B))y(I + tB))|_{t=0} = \text{Ad}(y^{-1})\theta(B) + B.
\]

p.492, l.\(-12\) REPLACE: \(\text{Sp}(\omega)\) BY: \(\text{Sp}(C^{2n}, \omega)\)

p.500, l.115 REPLACE:

and distinct regular homomorphisms

BY:

and regular homomorphisms

p.500, l.\(-10\) REPLACE:

Then we have distinct regular characters

BY:

Then we have regular characters

p.500, l.\(-10\) REPLACE: \(\cdots \supset V_r\) with \(\vdots \supset V_r \supset V_{r+1} = \{0\}\) with

p.501, l.5 REPLACE:

Given \(v \in V_r, x \in \mathcal{D}(G)\), and \(g \in G\) we have

BY:

If \(v \in V\) and \(\pi(x)v = \theta_r(x)v\) for all \(x \in \mathcal{D}(G)\), then

p.501, l.7 and l.8 REPLACE:

Thus \(\pi(g)v \in V_r\). Since \(\pi\) is an irreducible representation, this implies that \(V = V_r\). We conclude that \(r = 1\) and \(\pi(x) = \theta_1(x)I\) for all \(x \in \mathcal{D}(G)\).

BY:

Thus \(\pi(x)v = \theta_r(x)v\) for all \(v \in V\) and \(x \in \mathcal{D}(G)\), since the space of vectors with this property contains \(V_r \neq 0\) and is \(G\)-invariant. Write \(\theta_r = \theta\).

p.501, l.9, l.13, and l.14 REPLACE: \(\theta_1\) BY: \(\theta\)

p.502, l.8 REPLACE: element BY: elements
p.502, l.−5 REPLACE:

\[(\exp yX_0)g(\exp -yX_0) = t \exp[(t^{-\alpha} - 1)y + zX_0].\]

BY:

\[(\exp yX_0)g(\exp -yX_0) = t \exp[(t^{-\alpha} - 1)y + z]X_0].\]

p.504, l.13 to l.17 REPLACE:

Proof of Theorem 11.3.7  We may take \(G\) to be a closed subgroup of \(GL(n, \mathbb{C})\). Let \(X\) be the projective variety of full flags in \(\mathbb{C}^n\). Let \(B\) be a Borel subgroup of \(G\) of maximum dimension. Then Theorem 11.3.8 implies that the set \(Y = \{x \in X : bx = x \text{ for all } b \in B\}\) is nonempty. Fix \(y \in Y\) and set \(O = G \cdot y\). Set \(Z = \overline{O}\) (Zariski closure in \(X\)).

BY:

Proof of Theorem 11.3.7  Let \(B\) be a Borel subgroup of \(G\) of maximum dimension. By Theorem 11.1.1 there is a representation \((\pi, V)\) of \(G\) and a point \(y \in \mathbb{P}(V)\) so that \(B\) is the stabilizer of \(y\). Set \(X = \mathbb{P}(V)\) and \(O = G \cdot y \subset X\). Then \(G/B \cong O\) as a quasi-projective set. Set \(Z = \overline{O}\) (Zariski closure in \(X\)).

p.505, l.10 REPLACE:  \(y \cdot (gB) - ygB\)  BY:  \(y \cdot (gB) = ygB\)

p.505, l.15 REPLACE:  \(\phi_k(x) = x^k\)  BY:  \(\Phi_k(x) = x^k\)

p.505, l.16 REPLACE:  \(G(k) \subset G(k + 1)\)  BY:  \(G(2^k) \subset G(2^{k+1})\)

p.515, l.10 REPLACE:  Theorem A.3.4  BY:  Theorem A.3.3

p.527, l.−7 REPLACE:  Theorem A.3.4  BY:  Theorem A.3.3

p.532, l.15 to l. 19 (Exercise #1) REPLACE:

1. Let \(L\) be a reductive group, and set \(G = L \times L\). Let \(K = \{(g, g) : g \in L\}\) be the diagonal embedding of \(L\) in \(G\). Show that \((G, K)\) is a spherical pair. (HINT: The irreducible representations of \(G\) are of the form \(\pi = \sigma \otimes \mu\), where \(\sigma\) and \(\mu\) are irreducible representations of \(L\). Use Schur’s Lemma to show that the \(K\)-spherical representations of \(G\) are the representations \(\pi = \sigma \otimes \sigma^*\).)

BY:

1. Use Theorem 12.2.1 to show that the following spaces are multiplicity-free:

(a) \(G = GL(n) \times GL(k), X = M_{n,k}(\mathbb{C}), (g, h) \cdot x = gxh^{-1}\). (HINT: Lemma B.2.8.)

(b) \(G = GL(n), X = SM_n(\mathbb{C}), g \cdot x = gxg^t\). (HINT: Lemma B.2.9.)

(c) \(G = GL(n), X = AM_n(\mathbb{C}), g \cdot x = gxg^t\). (HINT: Lemma B.2.10.)

p.534, l.16 REPLACE:  \(\tau(g) = (g^{-t})^{-1}\)  BY:  \(\tau(g) = (\bar{g}^t)^{-1}\)

p.538, l.8 REPLACE:  Theorem A.3.4  BY:  Theorem A.3.3
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**p.540, l.23** REPLACE: note that \( l = n - 1 \) is odd. BY: note that \( l = 2n - 1 \) is odd.

**p.550, l.1** REPLACE:

**TYPE AII:** \( \{\varpi_2, \varpi_4, \ldots, \varpi_1\} \) \( (p = l/2) \),

BY:

**TYPE AII:** \( \{\varpi_2, \varpi_4, \ldots, \varpi_{l-1}\} \) \( (p = (l - 1)/2) \),

**p.558, l.13** DELETE: Then

**p.566, l.4 and l.5** REPLACE: \( T_{f,j} \) BY: \( T_{j,f} \)

**p.566, l.8** REPLACE: \( V(f) \neq 0 \) BY: \( V(I(f)) \neq \{0\} \)

**p.582, l.8 to l.17** REPLACE STATEMENT AND PROOF OF LEMMA A.1.3 BY:

**LEMMA A.1.3** An element \( b \in B \) is integral over \( A \) if and only if there exists a finitely-generated \( A \)-submodule \( C \subset B \) such that \( b \cdot C \subset C \).

**Proof.** Let \( b \) satisfy (A.1.2). Then \( A[b] = A \cdot 1 + A \cdot b + \cdots + A \cdot b^{n-1} \) is a finitely-generated \( A \)-submodule, so we may take \( C = A[b] \). Conversely, suppose \( C \) exists as stated and is generated by \( \{x_1, \ldots, x_n\} \) as an \( A \)-module. Since \( bx_i \in C \), there are elements \( a_{ij} \in A \) so that

\[
bx_i - \sum_{j=1}^{n} a_{ij} x_j = 0 \quad \text{for } i = 1, \ldots, n.
\]

Since \( x_i \neq 0 \) and \( B \) has no zero divisors, the determinant of the coefficient array of the \( x_i \) must vanish. This determinant is a monic polynomial in \( b \), with coefficients in \( A \). Hence \( b \) is integral over \( A \). \( \square \)

**p.582, l.20 to l.23** REPLACE:

The submodule \( A[b] \) of \( B \) is therefore also finitely generated, for any \( b \in B \), and hence \( b \) is integral over \( A \).

BY:

Now apply Lemma A.1.3 with \( C = B \).

**p.587, l.2** REPLACE: but \( f_1 \) not vanishing BY: but \( f_i \) not vanishing

**p.587, l.3** REPLACE: and \( X \neq X_1 \). BY: and \( X \neq X_i \).

**p.588, l.9** REPLACE: \( \tilde{v}(x)/f(x) = 0 \). BY: \( \tilde{v}(x)/f(x)^k = 0 \).

**p.592, l.1** REPLACE: Let \( \phi \in \text{Aff}(X) \). BY: Let \( \phi \in \text{Aff}(Y) \).

**p.592, l.6** REPLACE: all \( \phi \in \text{Aff}(X) \). BY: all \( \phi \in \text{Aff}(Y) \).
**p.592, l.14 to l.10** REPLACE:

every $\phi \in \text{Hom}(A, C)^a$ extends to $\psi \in \text{Hom}(B, C)^b$.

Proof. We start with the case $B = A[u]$ for some element $u \in B$. Let $b = f(u)$ be given, where

$$f(X) = a_nX^n + \cdots + a_0, \quad a_i \in A.$$ 

**p.593, l.3** REPLACE:

element $a = a_mc_0$ has the desired property in this case.

**p.593, l.5 and l.6** REPLACE: $q(X)$ BY: $h(X)$

**p.599, l.15 and l.16** REPLACE:

a map $x \mapsto L_x$ from $X$ to $T(X)_x$

**p.601, l.13 to l.8** DELETE: Statement and proof of Corollary A.3.3

**p.601, l.7** REPLACE: Theorem A.3.4 BY: Theorem A.3.3

**p.602, l.1** REPLACE: Lemma A.3.5 BY: Lemma A.3.4

**p.603, l.9** REPLACE: Theorem A.3.4 BY: Theorem A.3.3

**p.602, l.9** REPLACE: Lemma A.3.5 BY: Lemma A.3.4

**p.602, l.12** REPLACE: Corollary A.3.6 BY: Corollary A.3.5

**p.602, l.9** REPLACE: Lemma A.3.5 BY: Lemma A.3.4

**p.603, l.8** REPLACE: Theorem A.3.4 BY: Theorem A.3.3

**p.604, l.12 to l.25** DELETE Exercises A.3.5 and replace by:

**Proposition A.3.6** Let $\varphi : X \to Y$ be a dominant regular map of irreducible affine algebraic sets. For $y \in Y$ let $F_y = \varphi^{-1}\{y\}$. Then there is a nonempty open set $U \subset X$ such that $\dim X = \dim Y + \dim F_{\varphi(x)}$ and $\dim F_{\varphi(x)} = \dim \text{Ker}(d\varphi_x)$ for all $x \in U$. 

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Proof. Let $d = \dim X - \dim Y$, $S = \varphi^*\text{Aff}(Y)$, and $R = \text{Aff}(X)$. Set $k = \text{Quot}(S)$ and let $B \subseteq \text{Quot}(R)$ be the subalgebra generated by $k$ and $R$ (the rational functions on $X$ with denominators in $S \setminus \{0\}$). Since $B$ has transcendence degree $d$ over $k$, Lemma A.1.17 furnishes an algebraically independent set $\{f_1, \ldots, f_d\} \subseteq R$ such that $B$ is integral over $k[f_1, \ldots, f_d]$. Taking the common denominator of a set of generators of the algebra $B$, we obtain $f = \varphi^*g \in S$ such that $R_f$ is integral over $S_f[f_1, \ldots, f_d]$, where $R_f = \text{Aff}(X^f)$ and $S_f = \varphi^*\text{Aff}(Y^g)$. By Theorem A.2.5 we can take $g$ so that $\varphi(Y^g) = X^f$.

Define $\psi : X^f \rightarrow Y^g \times \mathbb{C}^d$ by $\psi(x) = (\varphi(x), f_1(x), \ldots, f_d(x))$. Then $\psi^*\text{Aff}(Y^g \times \mathbb{C}^d) = S_f[f_1, \ldots, f_d]$, and hence $\text{Aff}(X^f)$ is integral over $\psi^*\text{Aff}(Y^g \times \mathbb{C}^d)$. By Theorem A.2.5 every homomorphism from $S_f[f_1, \ldots, f_d]$ to $\mathbb{C}$ extends to a homomorphism from $R_f$ to $\mathbb{C}$. Hence $\psi$ is surjective. Let $\pi : Y^g \times \mathbb{C}^d \rightarrow Y^g$ by $\pi(y, z) = y$. Then $\varphi = \pi \circ \psi$ and $F_y = \psi^{-1}(\{y\} \times \mathbb{C}^d)$. If $W$ is any irreducible component of $F_y$ then $\text{Aff}(W)$ is integral over $\psi^*\text{Aff}(\{y\} \times \mathbb{C}^d)$, and hence $\dim W = d$.

We have $d\varphi = d\pi \psi(x) d\psi_x$. By integrality, every derivation of $\text{Quot}(\psi^*(Y^g \times \mathbb{C}^d))$ extends uniquely to a derivation of $\text{Rat}(X^f)$, as in the proof of Theorem A.3.1. Hence $d\psi_x$ is bijective for $x$ in a nonempty dense open set $U$ by Lemma A.3.4. For such $x$, Ker$(d\varphi_x) = \text{Ker}(d\pi \psi(x))$ has dimension $d$. $\square$

p.606, l.–2 and l.–1 REPLACE:

$(x, y) \mapsto x^t y$, where $x^t$ is the transpose of $x$.

BY:

$(x, y) \mapsto xy^t$, where $y^t$ is the transpose of $y$.

p.607, l.8, l.12, and l.–3 REPLACE: $x^t y$ BY: $xy^t$

p.607, l.–5 REPLACE: $x^t x$ BY: $xx^t$

p.609, l.–4 to l.–1 REPLACE:

Corollary A.4.6 Let $X$ be a quasiprojective algebraic set and $\phi : X \rightarrow X$ a regular map. Then the fixed-point set $\{x \in X : \phi(x) = x\}$ is closed in $X$.

Proof. The fixed-point set of $\phi$ is the intersection of the closed sets $\Gamma_{\phi}$ and $\Delta$, where $\Delta$ is the diagonal in $X \times X$. $\square$

BY:

Corollary A.4.6 Let $X, Y$ be quasiprojective algebraic sets and $\phi : X \times Y \rightarrow X$ a regular map. Then $\{(x, y) \in X \times Y : \phi(x, y) = x\}$ is closed in $X \times Y$.

Proof. Use the same argument as for Proposition A.4.5. $\square$

p.664, l.8, l.9, and l.10 REPLACE:

$d\varphi : \text{Lie}(G) \rightarrow \text{Lie}(H)$ by $d\varphi(X)_1 = d\varphi_1(X)_1$. 
The content of the following result is that $d\varphi$ is a Lie algebra homomorphism.

BY:

$d\varphi : \text{Lie}(G) \to \text{Lie}(H)$ by $d\varphi(X)_1 = d\varphi_1(X_1)$.

**p.664, l.12 REPLACE:**

Proof. Use the same argument as in Theorem 1.2.10 □

**BY:**

Proof. If $f \in C^\infty(H)$ then $X(f \circ \varphi) = (d\varphi(X)f) \circ \varphi$ by the left-invariance of $X$. Hence $[X,Y](f \circ \varphi) = ([d\varphi(X),d\varphi(Y)]f) \circ \varphi$. This implies that $d\varphi([X,Y])_1 = ([d\varphi(X),d\varphi(Y)])_1$. □

**p.664, l.17 REPLACE:**

Thus Lemma D.2.5 implies that

**BY:**

Thus Theorem D.2.3 implies that