Using Technology Offstage in Linear Algebra Courses

(Project NExT Panel–Knoxville MathFest)

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Reasons to use state-of-the-art technology in teaching linear algebra

Pragmatic:

- All applied science fields use computations with linear algebra models
- Experience with linear algebra software is needed for math-science-engineering careers

Pedagogical:

Contrast

- Calculus (*classical mechanics*):
  (a) multiplication of functions is commutative
  (b) can divide by a nonzero number

with

- Matrix Algebra (*quantum mechanics*):
  (a) multiplication of matrices is noncommutative
  (b) can't always divide by a nonzero matrix
- Students try to use rules from calculus to manipulate vectors and matrices.
- Students need *concept images* – not just *concept definitions* from axiomatic approach.

- Example: Using linear algebra software helps students to
  (a) visualize reduced row-echelon form R and positions of leading entries,
  (b) mentally carry out product of R with a column vector X of unknowns
  (c) visualize the positions of the unknowns in equation $RX = 0$.
- Students need *spatial symbol manipulation skills* for matrix calculations.

Theoretical:

- Linear algebra is *accessible* and *profound*
- Example: Algorithm for reduced row echelon form of a matrix
  (a) Individual steps easy using computer but requires choices – uniqueness not obvious
  (b) Computational exploration gives experimental evidence for uniqueness
  (c) Student acquires a concept image for the algorithm
  (d) Proof of uniqueness motivated by the student's experience
  (e) Uniqueness proof gives basis for column space

Rutgers Linear Algebra Labs

Used in three courses:

- Elementary Linear Algebra – required for math, computer science, chemistry students and elective for upper-level physics, engineering students

- Linear Algebra with Applications – first-year graduate engineering students

- Topics in Applied Algebra (emphasizes linear transformations, finite Fourier transform, finite wavelet transforms) – math, computer science, physics, and engineering students

Overview:

- Uses some T-codes from M.I.T. linear algebra course and some ideas from ATLAST project

- Revised over a six year development period

- Used by many instructors in the introductory course

Features

(a) Comprehensive

- MATLAB calculations illustrate all the major topics in the courses.
- Lecturer does not discuss MATLAB programming in class

(b) Individualized

- Student ID seeds random number generator – Matlab generates random matrices
- Calculate with random matrices to illustrate linear algebra concepts.
- Create diaries of sessions – insert comments and algebraic hand calculations.

(c) Include Applications

- Applied problems in each lab assignment
- More applications in topics course and graduate course

(d) Grading

- Lab assignments are done out of class — 25% of course grade.
- Details of MATLAB programming (such as m-files) are given
- Graders (upper-level undergraduates) write comments on student's paper.
- No electronic submission

(e) Effectiveness

- Labs reinforce the ideas presented in class — bridge gap between theory and computation
- Labs develop linear algebra *concept images* and *spatial symbol manipulation skills*
- Student course evaluations:
  - MATLAB cited as outstanding feature in all courses
  - In introductory course, agreement level 4 out of 5 to statements:

    "The MATLAB part of the course helped me understand linear algebra."

    "The MATLAB part of the course helped me on the exams."