Using Technology Offstage in Linear Algebra Courses

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March 25, 2006

Matlab Instructional Materials

- Introductory linear algebra course—required for math, computer science, and chemistry students and technical elective for upper-level engineering students

- Linear algebra course with applications for first-year graduate engineering students (mostly computer & electrical engineering)

- Second undergraduate course in applied linear algebra emphasizing linear transformations, finite Fourier transform, finite wavelet transform (math, computer science, and engineering students)

Features:

- Coordinated with the topics in the courses

- Students manipulate randomly-generated matrices to explore the linear algebra concepts presented in the lectures (prevents copying of lab assignments)

- Students use university computer labs outside of class— instructor does not need to know Matlab

Math 250: Intro to Linear Algebra

Text: Spence, Insel & Friedberg *Elementary Linear Algebra: A Matrix Approach*

LAB 1: Matrix and Vector Computations

- Creating matrices and vectors
- Matrix addition and matrix-vector multiplication
- Finding the reduced row echelon form of a matrix
- Leontief input-output economic model

LAB 2: Linear Equations and Matrix Algebra

- Solving linear equations by reduced row echelon form
- Forming linear combinations and testing for linear independence
- Matrix multiplication and its properties
- Adjacency Matrices for Communication Networks

LAB 3: LU Decomposition and Determinants

- LU decomposition of an invertible matrix A
- Solving $A\mathbf{x} = \mathbf{b}$ using the LU decomposition
- Comparison of computation time for Gaussian elimination vs. LU decomposition
- Properties of the determinant and calculation by the LU decomposition
- Geometric properties of matrices for rotations, dilations, and shears

LAB 4: General Solution to $A\mathbf{x} = \mathbf{b}$

- The column space and null space of A
- Particular and complete solution to $A\mathbf{x} = \mathbf{b}$
- Application to a traffic flow problem

LAB 5: Eigenvalues and Eigenvectors

- Geometric meaning of eigenvalues and eigenvectors
- Characteristic polynomial and eigenvalues
- Use of eigenvectors to diagonalize a matrix
- Steady-state eigenvector for a transition matrix
- Applications to Markov chains

LAB 6: Orthonormal Bases and Orthogonal Projections

- Norm and Inner product; orthogonal projection onto a line
- Gram-Schmidt Algorithm
- Orthogonal projection of a vector onto a subspace
- Best approximate solution to an inconsistent linear system
- Least squares fitting of curves to data points

Math 550: Linear Algebra & Applications

Text: Strang, *Linear Algebra and its Applications*

LAB 1: Gaussian Elimination, LU Factorization, and Solving $A\mathbf{x} = \mathbf{b}$

LAB 2: Orthogonal Projections, the Four Fundamental Subspaces, QR Factorization, and Inconsistent Linear Systems

LAB 3: Determinants, Eigenvalues, and Eigenvectors

Similar to the Math 250 labs covering these topics—additional items are:

- The four fundamental subspaces associated with a matrix
- QR matrix factorization
- Determinant formula for the inverse of a matrix

LAB 4: Unitary Diagonalization of Matrices, QR Algorithm, Finite/Fast Fourier Transform

- Diagonalization of hermitian and normal matrices by unitary matrices
- QR algorithm for fast computation of eigenvalues
- Fourier matrix and Fourier basis for $\mathbf{C}^n$
- Diagonalization of circulant matrices by the Fourier matrix
- Fast Fourier transform

LAB 5: Positive-Definite Matrices, Cholesky Factorization, Singular Value Decomposition

- Tests for positive-definiteness of a real symmetric matrix
- Cholesky factorization of a positive-definite matrix
- Singular value decomposition (SVD)
- Digital image processing using the SVD

Math 357: Topics in Applied Algebra

Prerequisites: Introductory linear algebra and multivariable calculus

Texts:

Leon: *Linear Algebra with Applications* (ch. 3-5)

Jensen & la Cour-Harbo: *Ripples in Mathematics: The Discrete Wavelet Transform*

Goodman: *Discrete Fourier Transform and Wavelet Transforms* (lecture notes)

LAB 1: Visualizing Linear Transformations

- Encoding two-dimensional polygonal figures as matrices
- Rotations, dilations, and shearing transformations
- Homogeneous coordinates and affine transformations

LAB 2: Convolution and Finite Fourier Transform

- Fourier matrix and Fourier basis for $\mathbf{C}^n$
- Applications of the finite Fourier transform—touch-tone dialing and the spectrum of a train whistle
- Discrete periodic signals and convolution—diagonalization of circulant matrices
- Fast Fourier transform

LAB 3: Haar Wavelet Transform

- Haar wavelet basis, the Haar analysis matrix, and the Haar synthesis matrix
- Fast Haar transform implemented by lifting
- Applications of the Haar transform—analysis of synthetic signals, filtering and compressing a noisy signal

LAB 4: Implementation of Wavelet Transforms

- The CDF(2,2) wavelet transform (matrix version)
- The Daub4 wavelet transform (matrix version)
- Fast Daub4 wavelet transform
- Multiresolution analysis using the CDF(2,2) wavelet transform—analysis of synthetic signals, filtering and compressing a noisy signal

LAB 5: Wavelet Analysis of Two-Dimensional Images

- Two-dimensional discrete wavelet transform
- Multiscale analysis of images
- Fast two-dimensional wavelet transform
- Denoising and compressing images by wavelet methods