

Workshop – Math 311 – February 9

1. Suppose α is a real number and E is a non-empty set of real numbers. Define

$$\alpha E = \{\alpha x : x \in E\} .$$

How is $\sup \alpha E$ related to $\sup E$ and $\inf E$? Conjecture and then prove your claim.

2. Prove that if A is a bounded above set of real numbers and $\alpha = \sup A$ then there exists a sequence $\{a_n\}$ of elements of A (i.e. $a_n \in A$ for all n) such that $\{a_n\}$ converges to α .
3. (a) Let $\{a_n\}$ and $\{b_n\}$ be any two convergent sequences of real numbers satisfying $a_n < b_n$ for all $n \in \mathbb{N}$. Prove that

$$\lim_{n \rightarrow \infty} a_n \leq \lim_{n \rightarrow \infty} b_n .$$

- (b) Give an example of two convergent sequences $\{a_n\}$ and $\{b_n\}$ satisfying $a_n < b_n$ or all $n \in \mathbb{N}$ such that

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n .$$

4. Show that the sequence defined recursively by

$$x_1 = 1 \quad \text{and} \quad x_{n+1} = \frac{1}{2 + x_n^2}, \text{ for all } n \geq 1$$

is Cauchy and hence convergent. Find its limit.