REVIEW QUESTIONS

Note: You don't have to answer these question verbally. Just make sure you know the answers in your mind and make sure you are capable on solving example problems.

1. First order ODEs

(1) What is a direction field for an ODE? How to draw it? What is an integral curve? How to sketch it? What is the relationship among ODE, solutions of ODE, direction field and integral curves?

(2) What is an initial value problem? What is the difference between a solution of an IVP to the general solution of ODE?

(3) How to find the order of an ODE? How to determine whether an ODE is linear or nonlinear?

(4) What is the standard form of a first-order linear ODE? How to find its integrating factor? How to check if you have the correct integrating factor? How to use the integrating factor to solve the first-order linear ODE? How to check if you have the correct solution?

(5) What does it mean for an ODE to be separable? How to solve it? How to find the explicit solution for such ODE with initial conditions? How to find the interval of existence?

(6) When does a first-order ODE have a unique solution? What are the conditions? How to find the the regions where the solution exists uniquely? Also answer these questions for a second-order linear homogeneous ODE.

(7) What is an autonomous ODE? What is its equilibrium solution? What does it mean for an equilibrium solution to be stable from above, stable from below, unstable from above, unstable from below? And what does it mean for an equilibrium solution to be stable, semistable and unstable?

(8) What is an exact ODE? How to solve an exact ODE? What can you do when you find your ODE not exact?

(9) What is the idea of numerical method? How to perform Euler's method? What is the local truncation error and the global truncation error respectively for Euler's method, improved Euler's method and Runge-Kutta's method?

2. Second and Higher order linear ODEs

(1) What is a characteristic equation? What can its root be? How to use that to solve a second-order linear homogeneous ODE whose coefficients are constant?

(2) What does it mean for two functions to be independent? Show an example where two functions are not independent. What is a fundamental set of solutions for a second-order linear homogeneous ODE?
(3) What is the principle of superposition? What can you conclude if you have a fundamental set of solutions for a second-order linear homogeneous ODE?
(4) How to analyze the long-term behavior of the solution to an IVP? For a parameterized IVP, how to find the critical point of the parameter when the behavior of the solution changes?
(5) For a second order linear homogeneous ODE with generic coefficients, how to use the knowledge of one particular solution to get the general solution?
(6) What does the method of undetermined coefficients do? When can you apply it? What are the first try templates if the right hand side of the nonhomogeneous linear ODE is a
   - constant function
   - polynomial function
   - exponential function
   - trigonometric function
   - product of a polynomial function and an exponential function
   - product of a polynomial function and a trigonometric function
   - product of a trigonometric function and an exponential function
   - product of a polynomial function, an exponential function and a trigonometric function
   - sums of different functions above
(7) When applying the method of undetermined coefficients, how to modify your template to the final template?
(8) For a second order linear ODE with the knowledge of the complementary solution, how to find a particular solution using the variation of parameters?
(9) What types of vibrations do we have? What are the corresponding equations of motion?
(10) For free vibrations, what do undamped, underdamped, critically-damped and overdamped mean? How do the characteristic roots look like in each of the cases above?
(11) For forced vibrations, when does resonance happen? When do the notions "transient solution" and "steady-state solution" make sense?
(12) When does the IVP $y^{(n)} + p_1(t)y^{(n-1)} + \cdots + p_n(t)y = g(t), y(0) = y_0, \cdots, y^{(n-1)}(0) = y_0^{(n-1)}$ has a unique solution?
(13) What is a fundamental set of solution? What does it mean for functions $y_1(t), \cdots, y_n(t)$ to be linearly dependent or linearly independent?
(14) What is a Wronskian and how is it related to linear dependency and independency? How to compute the Wronskian for a given set of functions?

(15) What is the structure of solutions for nonhomogeneous linear ODE? What is the complementary solution and what is a particular solution?

(16) For a homogeneous linear ODE with constant coefficients, what is its characteristic equation? How does its root determine the general solution? What happens when you have repeated root and how to deal with complex roots?

(17) How to apply the method of undetermined coefficients to solve higher order linear nonhomogeneous ODE with constant coefficients?

3. Systems of ODEs

(1) How to perform the multiplication of row vectors and column vectors? How about the multiplication of a matrix and a column vector?

(2) How to solve a system of linear equations? How to use the augmented matrix to proceed? What happens if the matrix of the (homogeneous or nonhomogeneous) system is nonsingular? What happens if the matrix of the (homogeneous or nonhomogeneous) system is singular? How many cases do we have to care about and how would the solutions look like?

(3) How to determine if a set of vectors is linear dependent or independent? If linear dependent, how to find a relation?

(4) How to find the eigenvalues and eigenvectors to a given matrix?

(5) What is the structure of the solutions to a (homogeneous) linear system of ODE? What is a fundamental set of solutions and how to verify if you have a fundamental set of solutions?

(6) How to solve a homogeneous linear system of ODE with constant coefficient? How to deal with the cases of distinct real eigenvalues, distinct complex eigenvalues and repeated real eigenvalues?

(7) How to draw the phase portraits for the linear system with distinct real eigenvalues and distinct complex eigenvalues?

(8) What is the (only) critical point for a nondegenerate linear system? How many types are there? What is the stability for each type?

(9) What are the critical points for a nonlinear system? How to find them?

(10) How to use linear systems to approximate a nonlinear system? How to draw the local phase portrait near each critical point? What information could you draw from the phase portrait?