1. Solve the following linear systems of equations

(a) $\begin{cases} x_1 - x_2 + x_3 &=& 2 \\ 2x_1 - x_2 - x_3 &=& 2 \\ -x_1 - 3x_2 + x_3 &=& -4 \end{cases}$

(b) $\begin{cases} x_1 - x_2 + x_3 &=& 1 \\ 2x_1 - x_2 - x_3 &=& 0 \\ 5x_1 - 3x_2 - x_3 &=& 2 \end{cases}$

(c) $\begin{cases} x_1 - x_2 + x_3 &=& 1 \\ 2x_1 - x_2 - x_3 &=& 0 \\ 5x_1 - 3x_2 - x_3 &=& 1 \end{cases}$

(d) $\begin{cases} x_1 - x_2 + x_3 &=& 1 \\ 2x_1 - 2x_2 + 2x_3 &=& 2 \\ -5x_1 + 5x_2 - 5x_3 &=& -5 \end{cases}$

2. Determine whether the members of the given set of vectors are linearly independent. If they are linearly dependent, find a linear relation among them.

(a) $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$

(b) $\vec{v_1} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}.$

(c) $\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$

(d) $\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \vec{v_4} = \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}.$

3. Find the eigenvalues and eigenvectors of the following matrices

(a) $\begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -1 \\ 5 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$