

HOMEWORK 20

1. Find the first four terms in each of two solutions y_1 and y_2 (unless the series terminates sooner) about the given point x_0 .

(a) $y'' - xy' - y = 0, x_0 = 1$

(b) $y'' - xy' - y = 0, x_0 = 0$

(c) $y'' + xy' + 2y = 0, x_0 = 0$

(d) $x^2y'' - x(x+2)y' + (x+2)y = 0, x_0 = 0$

(e) $(3 - x^2)y'' - 3xy' - y = 0, x_0 = 0$

Added 12:15PM: Problem 1d is set to be a bonus problem. It can still be solved using the techniques we talked about in class but just accidentally, due to the reason that x_0 is indeed a singular point (which does not belong to our current syllabus). Also Problem 1e was changed due to the same reason. Also in Problem 1d, the recurrence relation is indeed tricky to solve.

2. Determine a lower bound for the radius of convergence of series solutions about each given point x_0 for the given differential equation.

(a) $y'' + 4y' + 6xy = 0; x_0 = 0, x_0 = 4$

(b) $(x^2 - 2x - 3)y'' + xy + 4y = 0; x_0 = 4, x_0 = -4, x_0 = 0$

(c) $(1 + x^3)y'' + 4xy' + y = 0; x_0 = 0, x_0 = 2$

(d) $xy'' + y = 0; x_0 = 1$