Homework 16

1. A mass weighing 2 lb stretches a spring 6 in. Assuming there's no damping in the system. If the mass is pulled down an additional 3 in and then released, meanwhile is acted on by an additional oscillating force with amplitude 1 lb and phase 0. Find out the frequency of the force such that resonance happen, and determine the position at any time t by solving the system.

2. With all the data as above, suppose for now the system is subject to a damping force that is as strong as 2 lb when the velocity of the mass is 4 ft/s. Find the transient solution and the steady-state solution.

3. Find the general solution to the following ODEs

(a) $y'' + y = \tan t$

(b) $4y'' + y = 2\sec(t/2)$

(c) $x^2y'' - 2y = 3x^2 - 1, x > 0$

4. Knowing y_1 is a solution to the homogeneous ODE, find the general solution to the following ODEs

(a) $y_1 = x, x^2y'' - x(x+2)y' + (x+2)y(x) = 2x^2, x > 0$

(b) $y_1 = e^x, (1-x)y'' + xy' - y = 2(1-x)$

5. (Bonus) Apply the variation of parameters to the first order linear ODE

$$y' + p(t)y = g(t)$$

to obtain the formula of the general solution. (Hint: Solve the homogeneous ODE $y' + p(t)y = 0$ first to get the complementary solution Cy_1. Then set $Y(t) = u(t)y_1(t)$ and put in the ODE to find Y. So the general solution would then be $Cy_1 + Y$.