Math 244 **Name (Print):** ____________________
Summer 2015
Final Exam
7/17/14
Time Limit: 180 Minutes

This exam contains 17 pages (including this cover page) and 14 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	15	
3	20	
4	20	
5	10	
6	10	
7	10	
8	10	
9	15	
10	10	
11	20	
12	15	
13	15	
14	20	
Total:	200	

1. (10 points) Solve the initial value problem

$$y' - 2y = t^2, y(0) = 0$$

$$\mu(t) = e^{-2t}$$

$$y(t) = \frac{\int e^{-2t} \cdot t^2 dt + C}{e^{-2t}} = \frac{-\frac{1}{2}e^{-2t}t^2 + \int e^{-2t} \cdot t\, dt + C}{e^{-2t}}$$

$$= e^{2t}\left(-\frac{1}{2}e^{-2t} - \frac{1}{2}te^{-2t} + \int \frac{1}{2}e^{-2t} dt + C\right)$$

$$= e^{2t}\left(-\frac{1}{2}e^{-2t} - \frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t} + C\right)$$

$$= -\frac{1}{2}t^2 - \frac{1}{2}t - \frac{1}{4} + Ce^{2t}$$

$$y(0) = 0 \Rightarrow -\frac{1}{4} + C = 0 \Rightarrow C = \frac{1}{4}$$

Sol'n: $y = -\frac{1}{2}t^2 - \frac{1}{2}t - \frac{1}{4} + \frac{1}{4}e^{-2t}$

Integrating factor: 2pt. Integration by parts: 6pt. Solution: 2pt.

2. (15 points) Find the *explicit* solution to the following IVP

$$(1 + e^x)yy' = e^x, y(0) = 1$$

and specify the interval of existence

$$(1+e^x)\, y \frac{dy}{dx} = e^x$$

$$y\,dy = \frac{e^x}{1+e^x}\,dx$$

$$\frac{1}{2}y^2 = \ln(1+e^x) + C$$

$$y(0)=1 \Rightarrow \frac{1}{2} = \ln(1+1) + C \Rightarrow C = \frac{1}{2} - \ln 2$$

$$\frac{1}{2}y^2 = \ln(1+e^x) + \frac{1}{2} - \ln 2$$

$$y = \pm\sqrt{2\ln(1+e^x) + 1 - 2\ln 2}$$

Since $y(0) > 0$, should take positive branch

$$y = \sqrt{2\ln\frac{1+e^x}{2} + 1}$$

The sol'n makes sense when $2\ln\frac{1+e^x}{2} + 1 \geq 0$

$$\ln\frac{1+e^x}{2} \geq -\frac{1}{2}$$

$$\frac{1+e^x}{2} \geq e^{-\frac{1}{2}}$$

$$e^x \geq 2e^{-\frac{1}{2}} - 1$$

$$x \geq \ln\left(2e^{-\frac{1}{2}} - 1\right)$$

Interval of existence : $\left(\ln 2e^{-\frac{1}{2}} - 1, \infty\right)$

Integration 8 pt.

Explicit solution : 4 pt.

Interval of existence: 3pt

3. (20 points) Find the general solution to the ODE

$$\frac{y}{x} + (y^3 - \ln x)y' = 0$$

Hint: you have to find an integrating factor to get an exact ODE.

$M = \frac{y}{x}$. $N = y^3 - \ln x$

$M_y - N_x = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$

$\frac{M_y - N_x}{-M} = \frac{\frac{2}{x}}{-\frac{y}{x}} = -\frac{2}{y}$ indep. of x

$\frac{d\mu}{\mu} = -\frac{2}{y} dy \Rightarrow \ln \mu = -2 \ln y \Rightarrow \mu = y^{-2}$

$\frac{1}{xy} + \left(y - \frac{\ln x}{y^2}\right) y' = 0$, $M = \frac{1}{xy}$. $N = y - \frac{\ln x}{y^2}$

$M_y = -\frac{1}{xy^2}$, $N_x = -\frac{1}{xy^2}$, exact ✓

$\psi(x,y) = \int M\,dx + h(y) = \int \frac{1}{xy} dx + h(y) = \frac{\ln x}{y} + h(y)$

$\frac{\partial \psi}{\partial y} = -\frac{\ln x}{y^2} + h'(y) = y - \frac{\ln x}{y^2} \Rightarrow h'(y) = y \Rightarrow h(y) = \frac{1}{2} y^2$

Solution: $\frac{\ln x}{y} + \frac{1}{2} y^2 = C$

Integrating factor: 8pt.

Solving the exact ODE: 10pt

Final Answer: 2pt

4. (a) (10 points) Find the interval of existence for the solution to the following IVP

$$(\sin 2t)y'' + (\tan 4t)y' + \frac{y}{t} = 0, y(\pi/4) = 0, y'(\pi/4) = 1$$

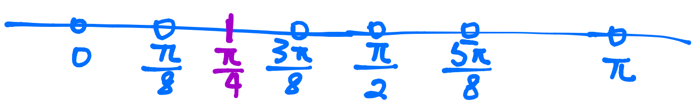
Std. form: $y'' + \frac{\tan 4t}{\sin 2t} y' + \frac{1}{t \sin 2t} y = 0$

$\frac{\tan 4t}{\sin 2t} = \frac{\sin 4t}{\sin 2t \cos 4t}$ not defined when $\sin 2t = 0$ or $\cos 4t = 0$

$\frac{1}{t \sin 2t}$ is not defined when $t = 0$ or $\sin 2t = 0$

$\sin 2t = 0 \Rightarrow 2t = k\pi,\ k = 0, \pm 1, \pm 2, \cdots; \Rightarrow t = \frac{k\pi}{2},\ k = 0, \pm 1, \pm 2, \cdots$

$\cos 4t = 0 \Rightarrow 4t = k\pi + \frac{\pi}{2},\ k = 0, \pm 1, \pm 2, \cdots \Rightarrow t = \frac{k\pi}{4} + \frac{\pi}{8},\ k = 0, \pm 1, \pm 2, \cdots$

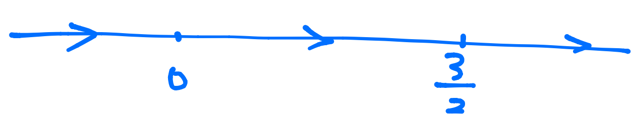


$\Rightarrow$ Interval of existence: $\left(\frac{\pi}{8}, \frac{3\pi}{8}\right)$

(b) (10 points) Find all the equilibrium solutions and draw the phase line of the autonomous ODE

$$y' = y^2(3 - 2y)^2$$

$y^2(3-2y)^2 = 0 \Rightarrow y = 0$ or $y = \frac{3}{2}$



(a) Singular points: 8pt. Interval: 2pt.

(b) Equilibriums and arrows: 2pt each.

5. (10 points) For the parameterized IVP

$$y'' + 6y' + 9y = 0, y(0) = 2, y'(0) = \alpha,$$

determine the critical point of α when the long-term behavior of the solution changes from eventually positive to eventually negative.

Char. eqn: $r^2 + 6r + 9 = 0 \Rightarrow r = -3, -3$

Gen. sol'n: $y = C_1 e^{-3t} + C_2 t e^{-3t}$

$y(0) = 2 \Rightarrow C_1 = 2$

$y'(0) = \alpha \Rightarrow -3C_1 + C_2 = \alpha \Rightarrow C_2 = \alpha + 6$

$y = 2e^{-3t} + (\alpha+6) t e^{-3t}$

$t \to \infty$, $(\alpha+6)te^{-3t}$ dominates

$\alpha + 6 > 0$, $y \to 0^+$

$\alpha + 6 < 0$, $y \to 0^-$

$\Rightarrow$ When $\alpha + 6 = 0$, behavior changes

Critical value of α : -6

Gen. sol'n: 4pt Solving C_1, C_2 : 2pt. Asymptotic Analysis: 4pt.

6. (10 points) Knowing that

$$y_1 = x$$

is a solution to the ODE

$$x^2(\ln x - 1)y'' - xy' + y = 0,$$

find the general solution.
Hint: For the ODE $y'' + p(x)y' + q(x)y = 0$, set $y_2(x) = u(x)y_1(x)$, then $u(x)$ should satisfy $y_1u'' + (2y_1' + py_1)u' = 0$.

Std. form: $y'' - \frac{1}{x(\ln x - 1)} y' + \frac{1}{x^2(\ln x - 1)} y = 0$

ODE of u: $x \cdot u'' + \left(2 - \frac{1}{x(\ln x - 1)} \cdot x\right) u' = 0$

$$\frac{u''}{u'} = -\frac{1}{x}\left(2 - \frac{1}{\ln x - 1}\right)$$

$$\ln u' = -2\ln x + \int \frac{1}{x} \cdot \frac{1}{\ln x - 1}\, dx$$

$$= -2\ln x + \int \frac{d \ln x}{\ln x - 1}$$

$$= -2\ln x + \ln(\ln x - 1)$$

$$\Rightarrow u' = x^{-2} \cdot (\ln x - 1)$$

$$u = \int \frac{\ln x - 1}{x^2}\, dx = \int (\ln x - 1) \cdot \frac{1}{x^2}\, dx$$

$$= -\frac{1}{x}(\ln x - 1) + \int \frac{1}{x} \cdot \frac{1}{x}\, dx$$

$$= -\frac{1}{x}(\ln x - 1) - \frac{1}{x} = -\frac{1}{x}\ln x$$

$$\Rightarrow y_2 = -\frac{1}{x}\ln x \cdot x = -\ln x$$

Gen. sol'n: $y = C_1 x + C_2(-\ln x) = C_1 x + C_2 \ln x$

Formulating the ODE: 3pt.

Solve for u': 3pt.

Integrating u': 3pt.

Final Ans: 1 pt.

7. (10 points) Find the general solution to the ODE

$$x^2y'' - xy' = 3x^3$$

Hint: For the ODE $y'' + p(x)y' + q(x)y = g(x)$, knowing the complementary solution $y_c(x) = C_1y_1(x) + C_2y_2(x)$, a particular solution can be obtained by the formula

$$Y = y_1(x)\int \frac{-y_2 g}{W(y_1, y_2)}dx + y_2(x)\int \frac{y_1 g}{W(y_1, y_2)}dx$$

$x^2y'' - xy' = 0$

Char. eqn: $r(r-1) - r = r^2 - 2r = 0 \Rightarrow r = 0,\ r = 2$

Comp. sol'n: $y = C_1 + C_2x^2$

$y_1 = 1$, $y_2 = x^2$; $W(y_1, y_2) = \begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix} = 2x$

Std. form: $y'' - \frac{1}{x}y' = 3x \Rightarrow g(x) = 3x$

$Y = 1 \cdot \int \frac{-x^2 \cdot 3x}{2x}dx + x^2 \cdot \int \frac{1 \cdot 3x}{2x}dx$

$= -\frac{1}{2}x^3 + \frac{3}{2}x^3 = x^3$

Gen. sol'n: $y = C_1 + C_2x^2 + x^3$

Comp. sol'n: 4pt. Particular sol'n: 5pt. Final answer: 1pt.

8. (10 points) Find the general solution to the ODE

$$y'' + 4y = 2\sin 2t - 3\cos 2t + 1$$

Comp. sol'n: $y_c = C_1 \cos 2t + C_2 \sin 2t$

Y_1 solves $y'' + 4y = 2\sin 2t - 3\cos 2t$

exp. coeff. $= 2i \Rightarrow$ fails once

$Y_1 = At\cos 2t + Bt\sin 2t$

$Y_1'' = -4A\sin 2t + 4B\cos 2t - 4At\cos 2t - 4Bt\sin 2t$

$Y_1'' + 4Y_1 = -4A\sin 2t + 4B\cos 2t = 2\sin 2t - 3\cos 2t$

$\Rightarrow A = -\frac{1}{2},\ B = -\frac{3}{4} \Rightarrow Y_1 = -\frac{1}{2}t\cos 2t - \frac{3}{4}t\sin 2t$

Y_2 solves $y'' + 4y = 1 \Rightarrow Y_2 = \frac{1}{4}$

Gen. sol'n: $y = C_1\cos 2t + C_2\sin 2t - \frac{1}{2}t\cos 2t - \frac{3}{4}t\sin 2t + \frac{1}{4}$

Comp. sol'n: 2pt. Templates: 3pt. Computation: 4pt. Sol'n: 1pt.

9. (15 points) Find the general solution to the ODE

$$y''' + y'' + y' + y = te^t$$

Char. eqn. $r^3 + r^2 + r + 1 = 0 \Rightarrow (r+1)(r^2+1) = 0 \Rightarrow r = -1, i, -i$

Comp. sol'n: $y_c = C_1 e^{-t} + C_2 \cos t + C_3 \sin t$

$Y = (At+B)e^t$

$Y' = (At + B + A)e^t$

$Y'' = (At + B + 2A)e^t$

$Y''' = (At + B + 3A)e^t$

$Y''' + Y'' + Y' + Y = (4At + 4B + 6A)e^t = te^t$

$\Rightarrow 4A = 1,\ 4B + 6A = 0 \Rightarrow A = \frac{1}{4},\ B = -\frac{3}{2}A = -\frac{3}{8}$

$Y = \left(\frac{1}{4}t - \frac{3}{8}\right)e^t$

$y = C_1 e^{-t} + C_2 \cos t + C_3 \sin t + \left(\frac{1}{4}t - \frac{3}{8}\right)e^t$

Complementary sol'n: 5pt.

Template: 3pt Computation 5pt. Final Sol'n: 2pt.

10. (10 points) Determine the final template for finding a particular solution to the following ODE

$$y'' - 2y' + 5y = te^t \cos 2t - t^2 e^t \sin 2t + 2te^t + 5\cos 2t + t^3$$

Warning: You are not asked to solve the coefficients!

Char. eqn: $r^2 - 2r + 5 = 0 \Rightarrow r = +1 \pm 2i$

$$Y = (A_0 t^3 + B_0 t^2 + C_0 t) e^t \cos 2t + (D_0 t^3 + E_0 t^2 + F_0 t) e^t \sin 2t$$
$$+ (A_1 t + B_1) e^t + A_2 \cos 2t + B_2 \sin 2t + A_3 t^3 + B_3 t^2 + C_3 t + D_3$$

4 summands, first try template for each summand 2pt.

modification by t. 2pt.

11. For the ODE

$$(3-x^2)y'' - 3xy' - y = 0, x_0 = 0$$

(a) (15 points) Find the first three nonzero terms of the series solution. (Hint: you only need to find up to a_5)

$$3\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^n - 3\sum_{n=1}^{\infty} n a_n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{m=0}^{\infty} 3(m+2)(m+1)a_{m+2}x^m - \sum_{m=2}^{\infty} m(m-1)a_m x^m - \sum_{m=1}^{\infty} 3m a_m x^m - \sum_{m=0}^{\infty} a_m x^m = 0$$

$$3\cdot 2\cdot 1\, a_2 + 3\cdot 3\cdot 2\, a_3 x + \sum_{m=2}^{\infty} 3(m+2)(m+1)a_{m+2}x^m$$

$$- \sum_{m=2}^{\infty} m(m-1)a_m x^m$$

$$-3a_1 x \qquad - \sum_{m=2}^{\infty} 3m a_m x^m$$

$$-a_0 \qquad -a_1 x \qquad - \sum_{m=2}^{\infty} a_m x^m$$

$$(6a_2 - a_0) + (18a_3 - 4a_1)x + \sum_{m=2}^{\infty}\left[3(m+2)(m+1)a_{m+2} - (m^2+2m+1)a_m\right]x^m = 0$$

$$\begin{cases} a_2 = \frac{1}{6}a_0 \\ a_3 = \frac{2}{9}a_1 \\ a_{m+2} = \frac{m+1}{3(m+2)}a_m, \ m \geq 2 \end{cases} \Rightarrow \quad \begin{aligned} a_4 &= \frac{3}{3\cdot 4}a_2 = \frac{1}{4}\cdot\frac{1}{6}a_0 = \frac{1}{24}a_0 \\ a_5 &= \frac{4}{3\cdot 5}a_3 = \frac{4}{15}\cdot\frac{2}{9}a_1 = \frac{8}{135}a_1 \end{aligned}$$

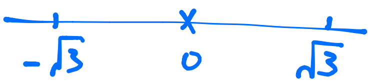
$$y = a_0\left(1 + \frac{1}{6}x^2 + \frac{1}{24}x^4 + \cdots\right) + a_1\left(x + \frac{2}{9}x^3 + \frac{8}{135}x^5 + \cdots\right)$$

The problem continues next page

(b) (5 points) Give a lower bound to the radius of convergence to the series.

$$y'' - \frac{3x}{3-x^2}y' - \frac{1}{3-x^2}y = 0$$

Singularities: $x = \pm\sqrt{3}$



Radius of convergence $\geq \sqrt{3}$

(a) Setting up the stage: 1 pt. Unifying exponents: 3 pt.
Unifying sums: 3 pt. Recurrence: 3pt. a_2, a_3, a_4, a_5 1pt each
Final Ans: 1 pt.

12. (15 points) Find the general solution to the linear system

$$\vec{x}' = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} \vec{x}$$

$$\begin{vmatrix} 1-\lambda & -1 & 4 \\ 3 & 2-\lambda & -1 \\ 2 & 1 & -1-\lambda \end{vmatrix} = (1-\lambda)\left[(2-\lambda)(-1-\lambda)+1\right] + 1\cdot(-3-3\lambda+2) + 4\cdot(3-4+2\lambda)$$

$$= (1-\lambda)\left[(\lambda^2-\lambda-2)+1\right] + (-1-3\lambda-4+8\lambda)$$

$$= (1-\lambda)(\lambda^2-\lambda-1) - 5(1-\lambda)$$

$$= (1-\lambda)(\lambda^2-\lambda-6) = 0 \Rightarrow \lambda = 1, -2, 3$$

$$\lambda = 1: \left[\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 3 & 1 & -1 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \vec{V}_1 = \begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix}$$

$$\lambda = -2: \left[\begin{array}{ccc|c} 3 & -1 & 4 & 0 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \vec{V}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = 3: \left[\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ 3 & -1 & -1 & 0 \\ 2 & 1 & -4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \vec{V}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Gen. sol'n: $\vec{x} = C_1 e^{t} \begin{bmatrix} 1 \\ -4 \\ -1 \end{bmatrix} + C_2 e^{-2t} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + C_3 e^{3t} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Eigenvalues: 6pt. Eigenvectors: 6pt. Sol'n: 3pt.

13. (15 points) Find the general solution to the linear system

$$\vec{x}' = \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix} \vec{x}$$

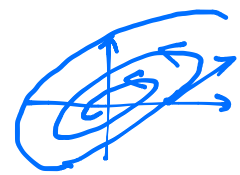
and draw the phase portrait

$$\begin{vmatrix} 1-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = \lambda^2+2\lambda-3+5 = \lambda^2+2\lambda+2 = 0 \Rightarrow \lambda = -1 \pm i$$

$$\lambda = -1+i : \left[\begin{array}{cc|c} 2-i & -1 & 0 \\ 5 & -2-i & 0 \end{array}\right] \Rightarrow \vec{v} = \begin{bmatrix} 1 \\ 2-i \end{bmatrix}$$

$$\vec{x} = C_1 e^{-t} \begin{bmatrix} \cos t \\ 2\cos t + \sin t \end{bmatrix} + C_2 \begin{bmatrix} \sin t \\ -\cos t + 2\sin t \end{bmatrix}$$

Tangent vector at $(1,0)$ is $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$



Eigenvalues: 3 pt. Eigenvector: 3 pt. Gen. sol'n: 5 pt. Phase portrait: 4 pt

14. (20 points) For the nonlinear system

$$\begin{cases} x' = y(2-x-y) \\ y' = -x-y-2xy \end{cases}$$

find all the critical points, give a classification to type and stability, and ~~draw the local phase portraits~~

Hint: there should be three critical points

$y(2-x-y)=0 \Rightarrow y=0$ or $y=2-x$

if $y=0$, then $-x-y-2xy=0 \Rightarrow x=0$

otherwise: $-x-y-2xy=0 \Rightarrow -2-2x(2-x) = 2x^2-4x-2=0$

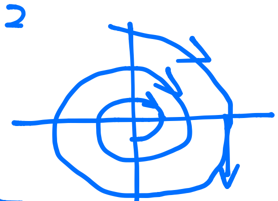
$\Rightarrow x = 1 \pm \sqrt{2} \Rightarrow y = 1 \mp \sqrt{2}$

Crit. pts: $(0,0)$, $(1+\sqrt{2}, 1-\sqrt{2})$, $(1-\sqrt{2}, 1+\sqrt{2})$

$$J(x,y) = \begin{bmatrix} -y & 2-x-2y \\ -1-2y & -1-2x \end{bmatrix}$$

Near $(0,0)$. $\vec{u}\,' = \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix} \vec{u}$. Eig-val: $\frac{-1 \pm \sqrt{7}i}{2}$

Negative real part $\Rightarrow$ spiral sink



Near $(1+\sqrt{2}, 1-\sqrt{2})$. $\vec{u}\,' = \begin{bmatrix} -1+\sqrt{2} & -1+\sqrt{2} \\ -3+2\sqrt{2} & -3-2\sqrt{2} \end{bmatrix}$

$$(-1+\sqrt{2}-\lambda)(-3-2\sqrt{2}-\lambda) - (-3+2\sqrt{2})(-1+\sqrt{2})$$

$$= \lambda^2 - \lambda(-1+\sqrt{2}-3-2\sqrt{2}) + (-1+\sqrt{2})(-3-2\sqrt{2}) - (-3+2\sqrt{2})(-1+\sqrt{2})$$

$$= \lambda^2 + (4+\sqrt{2})\lambda + 3-4-\sqrt{2} - (3-5\sqrt{2}+4)$$

You could use the next page to continue your solution.

You could use this page to continue solving Problem 14.

$$= \lambda^2 + (4+\sqrt{2})\lambda - 8 + 4\sqrt{2} = 0$$

Viète's theorem $\Rightarrow \lambda_1\lambda_2 = -8+4\sqrt{2} < 0 \Rightarrow \lambda_1, \lambda_2$ one positive one negative

$\Rightarrow$ saddle pt, unstable

Near $(1-\sqrt{2}, 1+\sqrt{2})$, $\vec{u}' = \begin{bmatrix} -1-\sqrt{2} & 2-1+\sqrt{2}-2-2\sqrt{2} \\ -1-2-2\sqrt{2} & -1-2+2\sqrt{2} \end{bmatrix} \vec{u}$

$$\vec{u}' = \begin{bmatrix} -1-\sqrt{2} & -1-\sqrt{2} \\ -3-2\sqrt{2} & -3+2\sqrt{2} \end{bmatrix} \vec{u}$$

$$(-1-\sqrt{2}-\lambda)(-3+2\sqrt{2}-\lambda) - (-1-\sqrt{2})(-3-2\sqrt{2})$$

$$= \lambda^2 + (4+\sqrt{2})\lambda + (3-4+\sqrt{2}) - (3+4+5\sqrt{2})$$

$$= \lambda^2 + (4+\sqrt{2})\lambda - 8 - 4\sqrt{2} = 0$$

$\lambda_1\lambda_2 < 0 \Rightarrow \lambda_1, \lambda_2$ one positive, one negative

$\Rightarrow$ saddle point. unstable.

Critical points: 6 pt

Jacobian matrices: 8 pt.

Eigenvalues and stability: 6 pt.