

Math 244
Summer 2015
Final Exam
7/17/15
Time Limit: 180 Minutes

Name (Print): _____

This exam contains 17 pages (including this cover page) and 14 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 10 | |
| 2 | 15 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 15 | |
| 10 | 10 | |
| 11 | 20 | |
| 12 | 15 | |
| 13 | 15 | |
| 14 | 20 | |
| Total: | 200 | |

1. (10 points) Solve the initial value problem

$$y' - 2y = t^2, y(0) = 0$$

2. (15 points) Find the *explicit* solution to the following IVP

$$(1 + e^x)yy' = e^x, y(0) = 1$$

and specify the interval of existence

3. (20 points) Find the general solution to the ODE

$$\frac{y}{x} + (y^3 - \ln x)y' = 0$$

Hint: you have to find an integrating factor to get an exact ODE.

4. (a) (10 points) Find the interval of existence for the solution to the following IVP

$$(\sin 2t)y'' + (\tan 4t)y' + \frac{y}{t} = 0, y(\pi/4) = 0, y'(\pi/4) = 1$$

- (b) (10 points) Find all the equilibrium solutions and draw the phase line of the autonomous ODE

$$y' = y^2(3 - 2y)^2$$

5. (10 points) For the parameterized IVP

$$y'' + 6y' + 9y = 0, y(0) = 2, y'(0) = \alpha,$$

determine the critical point of α when the long-term behavior of the solution changes from eventually positive to eventually negative.

6. (10 points) Knowing that

$$y_1 = x$$

is a solution to the ODE

$$x^2(\ln x - 1)y'' - xy' + y = 0,$$

find the general solution.

Hint: For the ODE $y'' + p(x)y' + q(x)y = 0$, set $y_2(x) = u(x)y_1(x)$, then $u(x)$ should satisfy $y_1u'' + (2y_1' + py_1)u' = 0$.

7. (10 points) Find the general solution to the ODE

$$x^2 y'' - xy' = 3x^3$$

Hint: For the ODE $y'' + p(x)y' + q(x)y = g(x)$, knowing the complementary solution $y_c(x) = C_1 y_1(x) + C_2 y_2(x)$, a particular solution can be obtained by the formula

$$Y = y_1(x) \int \frac{-y_2 g}{W(y_1, y_2)} dx + y_2(x) \int \frac{y_1 g}{W(y_1, y_2)} dx$$

8. (10 points) Find the general solution to the ODE

$$y'' + 4y = 2 \sin 2t - 3 \cos 2t + 1$$

9. (15 points) Find the general solution to the ODE

$$y''' + y'' + y' + y = te^t$$

10. (10 points) Determine the final template for finding a particular solution to the following ODE

$$y'' - 2y' + 5y = te^t \cos 2t - t^2 e^t \sin 2t + 2te^t + 5 \cos 2t + t^3$$

Warning: You are not asked to solve the coefficients!

11. For the ODE

$$(3 - x^2)y'' - 3xy' - y = 0, x_0 = 0$$

- (a) (15 points) Find the first three nonzero terms of the series solution. (Hint: you only need to find up to a_5)

The problem continues next page

(b) (5 points) Give a lower bound to the radius of convergence to the series.

12. (15 points) Find the general solution to the linear system

$$\vec{x}' = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} \vec{x}$$

13. (15 points) Find the general solution to the linear system

$$\vec{x}' = \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix} \vec{x}$$

and draw the phase portrait

14. (20 points) For the nonlinear system

$$\begin{cases} x' = y(2 - x - y) \\ y' = -x - y - 2xy \end{cases}$$

find all the critical points, give a classification to type and stability, and draw the local phase portraits.

Hint: there should be three critical points

You could use the next page to continue your solution.

You could use this page to continue solving Problem 14.