Corrigendum on “The direct and inverse scattering problems for partially coated obstacles” [Inverse Problems 17 (6), 2001, 1997-2015]

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1. Introduction

There is an error in the proof of Lemma 2.4 in [2] (pointed out in [3]). Specifically, in equation (29) of [2] we have incorrectly stated that the double layer operators K and K' are adjoint with respect to the L^2 inner product. However compact perturbations of each of these operators are adjoint and this is sufficient to correct the proof. In the following we shall provide a correction to the proof of Lemma 2.4 of [2].

2. Corrected Proof of Lemma 2.4 of [2].

Let us denote by $K_0 : H^{1/2+s}(\Gamma) \longrightarrow H^{1/2+s}(\Gamma)$ and $K_0' : H^{-1/2+s}(\Gamma) \longrightarrow H^{-1/2+s}(\Gamma)$ the operators

$$K_0\psi(x) = 2\int_\Gamma \psi(y)\frac{\partial}{\partial \nu_y}\Phi_0(x,y)ds_y \qquad K_0'\psi(x) = 2\int_\Gamma \psi(y)\frac{\partial}{\partial \nu_x}\Phi_0(x,y)ds_y$$

where $\Phi_0(x,y) := \frac{1}{2\pi}\ln\frac{1}{|x-y|}$. The operators $L_K := K - K_0$ and $L_{K'} := K' - K_0'$ are compact because they are both integral operators with continuous kernels. Furthermore K_0 and K_0' are adjoint with respect to the L^2-inner product. Then with the notation introduced in the proof of Lemma 2.4 in [2] we define the operator

$$A_0\psi = \begin{pmatrix} \left[(S_0\tilde{\psi}_D) - ik\lambda(S_0\tilde{\psi}_I) - K_0\tilde{\psi}_I\right]|_{\Gamma_D} \\ \\ \left[ik\lambda(S_0\tilde{\psi}_D) + k^2\lambda^2(S_0\tilde{\psi}_I) + (T_0\tilde{\psi}_I) + K_0'\tilde{\psi}_D - ik\lambda(K_0'\tilde{\psi}_I + K_0\tilde{\psi}_I)\right]|_{\Gamma_I} \end{pmatrix}$$

and

$$L_A\psi = \begin{pmatrix} (-L_S\tilde{\psi}_D + ik\lambda L_S\tilde{\psi}_I - L_K\tilde{\psi}_I)|_{\Gamma_D} \\ \\ (L_T\tilde{\psi}_I - ik\lambda L_S\tilde{\psi}_D - k^2\lambda^2 L_S\tilde{\psi}_I + L_{K'}\tilde{\psi}_D - ik\lambda(L_{K'}\tilde{\psi}_I + L_K\tilde{\psi}_I)|_{\Gamma_I} \end{pmatrix}$$

such that $A = (A_0 + L_A)$. With this definition, the operator $L_A : H \longrightarrow H^*$ is compact and $A_0 : H \longrightarrow H^*$ defines the sesquilinear form

$$\begin{aligned}\left\langle A_0\psi, \bar{\psi}\right\rangle_{H,H^*} &= (S_0\tilde{\psi}_D, \tilde{\psi}_D)_\Gamma + k^2\lambda^2(S_0\tilde{\psi}_I, \tilde{\psi}_I)_\Gamma \\ &- ik\lambda(S_0\tilde{\psi}_I, \tilde{\psi}_D)_\Gamma + ik\lambda(S_0\tilde{\psi}_D, \tilde{\psi}_I)_\Gamma \\ &- (K_0\tilde{\psi}_I, \tilde{\psi}_D)_{\Gamma_D} + (K_0'\tilde{\psi}_D, \tilde{\psi}_I)_\Gamma \\ &- ik\lambda((K_0 + K_0')\tilde{\psi}_I, \tilde{\psi}_I)_\Gamma + (T_0\tilde{\psi}_I, \tilde{\psi}_I)_\Gamma.\end{aligned} \tag{1}$$

Note that $(u, v)_\Gamma$ is the scalar product on $L^2(\Gamma)$ defined by $\int_\Gamma u\bar{v}\,ds$. Let us now take the real part of (1). From equation (24) in [2], we obtain

$$\begin{aligned}&\mathrm{Re}\left[(S_0\tilde{\psi}_D, \tilde{\psi}_D)_\Gamma + k^2\lambda^2(S_0\tilde{\psi}_I, \tilde{\psi}_I)_\Gamma - ik\lambda(S_0\tilde{\psi}_I, \tilde{\psi}_D)_\Gamma + ik\lambda(S_0\tilde{\psi}_D, \tilde{\psi}_I)_\Gamma\right] \\ &= \mathrm{Re}\left(S_0(\tilde{\psi}_D - ik\lambda\tilde{\psi}_I), (\tilde{\psi}_D - ik\lambda\tilde{\psi}_I)\right)_\Gamma \geq C_1\|\tilde{\psi}_D - ik\lambda\tilde{\psi}_I\|^2_{H^{-1/2}(\Gamma)}.\end{aligned} \tag{2}$$

Furthermore, since K_0 and K_0' are adjoint we have

$$\begin{aligned}&\mathrm{Re}\left[-(K_0\tilde{\psi}_I, \tilde{\psi}_D)_\Gamma + (K_0'\tilde{\psi}_D, \tilde{\psi}_I)_\Gamma\right] = \mathrm{Re}\left[-(K_0\tilde{\psi}_I, \tilde{\psi}_D)_\Gamma + (\tilde{\psi}_D, K_0'\tilde{\psi}_I)_\Gamma\right] \\ &= \mathrm{Re}\left[-(K_0\tilde{\psi}_I, \tilde{\psi}_D)_\Gamma + \overline{(K_0\tilde{\psi}_I, \tilde{\psi}_D)_\Gamma}\right] = 0,\end{aligned} \tag{3}$$

and

$$\mathrm{Re}\left[-ik\lambda((K_0 + K_0')\tilde{\psi}, \tilde{\psi})_\Gamma\right] = k\lambda\mathrm{Im}\left[(K_0\tilde{\psi}_I, \psi_I)_\Gamma + \overline{(K_0\tilde{\psi}_I, \tilde{\psi}_I)_\Gamma}\right] = 0.$$

Finally from equation (25) in [2] we have

$$\mathrm{Re}\left(T_0\tilde{\psi}_I, \tilde{\psi}_I\right)_\Gamma \geq C_2\|\tilde{\psi}_I\|^2_{H^{1/2}(\Gamma)}. \tag{4}$$

Combining (2), (3), (4) and (4) in (1) we conclude that

$$\mathrm{Re}\left\langle A_0\psi, \bar{\psi}\right\rangle_{H,H^*} \geq C_1\|\tilde{\psi}_D - ik\lambda\tilde{\psi}_I\|^2_{H^{-1/2}(\Gamma)} + C_2\|\tilde{\psi}_I\|^2_{H^{1/2}(\Gamma)} \geq C\|\psi\|^2_H,$$

for any $\psi \in \tilde{H}^{-1/2}(\Gamma_D) \times \tilde{H}^{1/2}(\Gamma_I)$ where we have used the arithmetic geometric mean inequality in deriving the last inequality. From this and the compactness of the operator L_A, we can now conclude that the operator $A = A_0 + L_A$ is Fredholm with index zero [4], and the proof of Lemma 2.4 continues as in our paper.

Remark 2.1 *Lemma 2.4 of [2] is only used to prove the well-posedness of the associated interior mixed boundary value problem (and by analogy the exterior boundary value problem: see Theorems 2.3 and 2.5 of [2]). An alternative proof of these well-posedness results, based on a variational approach, is given in Theorems 8.4 and Theorem 8.5 in [1]. The variational approach allows variable λ and hence extends the results of [2].*

References

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