All problems are worth an equal weight, even if some may be harder than others.

Write answers on a separate sheet, either typed or handwritten. No need to copy the problem statements. Staple separate sheets. Paperclips or folded corners don’t count!

Due at the beginning of lecture, as a physical copy (no electronic submission).

Circle your final answer, but make sure to include the work to get there.

It is OK to work with others, as long as you acknowledge them in your paper and write down your solutions in your own words. It is not OK to copy a solution from whatever source.

1. Bob initially has $1,000 in his bank account, which earns interest of 2% per year compounded continuously. He also deposits $100 every year into the account.
   a) Write an IVP for $S(t)$, the money in his account at time $t$ measured in years.
   b) Find the general solution to the differential equation three times, by three different methods: separation of variables, undetermined coefficients, and an integrating factor. Then find the value of $C$ in the general solution using the initial condition.
   c) How much time will it take for the account to reach $13,000?

Remark: Interest in real life is not actually compounded on a continuous basis, like your differential equation suggests, but rather once per month. The deposits in this problem are also made once per year, not continuously. But this turns out to be an accurate way of modeling the problem, and the formula you obtain can be used for easily computing earnings with interest.

2. Consider the equation
   $$y' = -y^2 + y + 2yt^2 + 2t - t^2 - t^4$$
   a) Verify that both $u = t^2$ and $v = t^2 + 1$ are solutions.
   b) Given the item above, what does the existence and uniqueness theorem imply about the solution $y(t)$ that satisfies $y(3) = 19/2$? Remember to verify that the theorem can be applied.

3. For each of the phase lines below, come up with a function $f$ that produces them in the differential equation $y' = f(y)$.

   a)
   b)

4. Can the derivative criterion be used to classify the equilibrium points of the equation below as sinks or sources? If not, classify them by plugging-in points and drawing the phase line.

   $$y' = y^3 \ln \left( \frac{y^2 + 5}{9} \right)$$
5. Let \( y' = (y - 1)(y - 3) \).
   a) Sketch the phase line, then based on it sketch the graphs of sample solutions.
   b) Solve the differential equation with \( y(0) = 2 \) and use calculus to sketch the graph of the solution. Compare to the corresponding graph from item (a).

6. Draw the bifurcation diagram of the following equations with respect to the parameter \( A \), and find bifurcation values if any exist.
   a) \( y' = (y + A)^2 - 25 \).
   b) \( y' = \cos y + A \).

7. Use undetermined coefficients to solve the following equations:
   a) \( y' = 3y + t^2 + 2 \).
   b) \( y' = -\frac{y}{3} + \cos 2t \).
   c) \( y' = y + 2t \sin 3t \). \text{Hint: For a particular solution here, try one of the form:}
   \[
   y_p(t) = A \sin 3t + B \cos 3t + Ct \sin 3t + Dt \cos 3t
   \]

8. Consider the equation
   \[ y' = y \cos t + \frac{1}{5} (1 - t \cos t) \]
   a) Verify that \( y = t/5 \) is a solution.
   b) Use the Extended Linearity Principle to find the general solution.

9. Use an integrating factor to solve the following equations:
   a) \( y' = 4t y + \frac{1}{t^3} \).
   b) \( y' = \frac{4t^4 + 4t}{t^4 + 2t^2} y + t^4 - 2t^2 \).
   c) \( y' = -5y + 3 \sin t \). \text{Remark: This is an example of when using undetermined coefficients would have been easier.}

10. An industrial container has a capacity of 144L, but initially it contains only 9L of water. Dissolved in this water are initially 10g of sugar. Water containing \( \frac{2}{3} \text{g/L} \) of sugar is pouring into it at a rate of 9L/min, but due to a hole in the container the contents are leaking at a rate of 3L/min. Assume the sugar is spread out evenly through the water in the container at all times. How many grams of sugar will there be in the container by the time when it’s full?
   \text{Notice this is a problem in which the volume of the contents does not stay constant.}