MATH 251 Quiz 4 - Feb 27 TA: Érik de Amorim

No calculators allowed (or needed). Write complete solutions, not just the answer.

Full name: Quiz Four

Name or nickname you'd like me to use: Q4

1. *(5 pts)* Use Lagrange multipliers to find the maximum and minimum values of the function $\boxed{f(x,y) = x^2y}$ over all points (x,y) that satisfy the constraint $\boxed{4x^2 + 9y^2 = 25}$.

$\nabla f = \langle 2xy, x^2 \rangle$, $\nabla g = \langle 8x, 18y \rangle$ $\quad g(x,y) = 4x^2 + 9y^2 - 25$

$$\begin{cases} 2xy = \lambda \cdot 8x \\ x^2 = \lambda \cdot 18y \\ 4x^2 + 9y^2 = 25 \end{cases}$$

Case 1: $x \neq 0$

First eq. becomes $2y = 8\lambda \Rightarrow \lambda = \frac{y}{4}$

Plug into second eq.: $x^2 = \frac{y}{4} \cdot 18y = \frac{9}{2}y^2$

Plug into third eq.: $4\left(\frac{9}{2}y^2\right) + 9y^2 = 25$

$27y^2 = 25$

$y = \pm\sqrt{\frac{25}{27}}$

Obtain x^2 using $x^2 = \frac{9}{2}y^2$:

$x^2 = \frac{9}{2} \cdot \frac{25}{27} = \frac{25}{6}$

Then the possible values of x^2y are $\pm \frac{25}{6}\sqrt{\frac{25}{27}}$

Case 2: $x = 0$

If $x = 0$, the only possible value of x^2y is (0)

The largest value found is $\boxed{\frac{25}{6}\sqrt{\frac{25}{27}}}$

The smallest is $\boxed{-\frac{25}{6}\sqrt{\frac{25}{27}}}$

ANOTHER PROBLEM ON THE BACK! →

2. *(5 pts)* For this problem, the function F is defined by

$$\boxed{F(x,y) = x^4 - 2x^2 + y^2 - 6y}$$

a) *(1 pt)* What is the gradient of F?

b) *(1 pt)* What are the critical points of F?

c) *(3 pts)* Use the second derivative test to classify each as a local maximum, local minimum or saddle point. If the test is inconclusive, simply write "inconclusive".

a) $\nabla F = \langle 4x^3 - 4x \, , \, 2y - 6 \rangle$

b) $4x^3 - 4x = 0$ $\qquad 2y - 6 = 0$

$x^3 = x$ $\qquad y = 3$

$x = 0, \pm 1$ $\qquad$ Critical pts: $(0,3)$, $(1,3)$, $(-1,3)$

c) $f_{xx}(x,y) = 12x^2 - 4$

$f_{yy}(x,y) = 2$

$f_{xy}(x,y) = 0$

$D(x,y) = 24x^2 - 8$

$D(0,3) = -8 \Rightarrow$ $(0,3)$ is a saddle point

$D(1,3) = 16$, $f_{xx}(1,3) = 8 \Rightarrow$ $(1,3)$ is a local minimum

$D(-1,3) = 16$, $f_{xx}(-1,3) = 8 \Rightarrow$ $(-1,3)$ is a local minimum

IF YOU NEED MORE PAPER, ASK THE TA.