MATH 251 Quiz 1 - Feb 06 TA: Érik de Amorim

No calculators allowed (or needed). Write complete solutions, not just the answer.

Full name: Solutions

Name or nickname you'd like me to use: Solutions Mc Solutionsface

1. (4 pts) Compute the angle between the following pairs of vectors. **Give your final answer in degrees**, for example $120°$ instead of $2\pi/3$ or $\cos^{-1}(-1/2)$.

   a) $\langle 3, \sqrt{3}\rangle$ and $\langle 1, \sqrt{3}\rangle$.

   b) $\langle -2, 8, 8\rangle$ and $\langle 4, 5, -4\rangle$.

a) $\vec{u} \cdot \vec{v} = 3 \cdot 1 + \sqrt{3} \cdot \sqrt{3} = 3 + 3 = 6$

$\|\vec{u}\| = \sqrt{3^2 + \sqrt{3}^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

$\|\vec{v}\| = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{1+3} = \sqrt{4} = 2$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|} = \frac{6}{2\sqrt{3} \cdot 2} = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{2 \cdot 3} = \frac{\sqrt{3}}{2}$$

$\Rightarrow \boxed{\theta = 30°}$

b) $\vec{u} \cdot \vec{v} = -2 \cdot 4 + 8 \cdot 5 + 8 \cdot (-4) = -8 + 40 - 32 = 0 \Rightarrow$ Orthogonal

$\Rightarrow \boxed{\theta = 90°}$

(The formula also works: $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|} = \dfrac{0}{\|\vec{u}\| \cdot \|\vec{v}\|} = 0 \Rightarrow \theta = 90°$)

2. (3 pts) Compute the following cross products:

a) $\langle 1,2,3\rangle \times \langle 5,1,2\rangle$.

b) $\langle 2,0,3\rangle \times \langle 4,0,6\rangle$.

a) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 5 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} \vec{k}$

$= (4-3)\vec{i} - (2-15)\vec{j} + (1-10)\vec{k} = 1\cdot\vec{i} + 13\vec{j} - 9\vec{k}$

$= \boxed{\langle 1, 13, -9\rangle}$

b) The vectors are parallel (multiple of each other), so the cross product is $\boxed{\vec{0}}$.

(The formula also works: $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 3 \\ 4 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ 0 & 6 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix} \vec{k} = 0\vec{i} + 0\vec{j} + 0\vec{k}$
$= \langle 0,0,0\rangle$)

3. (3 pts) If vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are such that $\|\boldsymbol{u}\| = 2$, $\|\boldsymbol{v}\| = 7$ and $\boldsymbol{u}\cdot\boldsymbol{v} = -4$, compute $\|\boldsymbol{u}-\boldsymbol{v}\|$. *Hint: use the fact that* $\|\boldsymbol{u}-\boldsymbol{v}\|^2 = (\boldsymbol{u}-\boldsymbol{v})\cdot(\boldsymbol{u}-\boldsymbol{v})$.

$\|\vec{u}-\vec{v}\|^2 = (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$= \|\vec{u}\|^2 - 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$

$= 4 - 2(-4) + 49$

$= 61$

$\Rightarrow \|\vec{u}-\vec{v}\| = \sqrt{61}$