Order 2 Rational Difference Equations Web Book

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Contents

0   Description of WebBook

Each section corresponds to a specific second order rational difference equation (with linear numerator and denominator). For each difference equation we chose 50 random parameter values. For each set of parameter values we first check that the equilibrium is LAS. If it isn't we stop there and move on to the next set of parameter values. If it is LAS then we start checking K values starting from 1 (with a maximum of 6) in order to try to prove GAS. If one of the K values works then we declare that the equilibrium is GAS, the "proof certificate" is the K value. In the case that there are multiple equilibrium points associated to a set of parameter values we check each equilibrium separately. The following sections contain a summary of the results, the parameter values and associated equilibrium are in the form [{params}, equilibrium].

1 $\dfrac{M^2}{x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{M^2}{x_{n-1}}$$

the equilibrium $\bar{x} = 0$ is not LAS.

2 $\dfrac{x_n}{x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{x_n}{x_{n-1}}$$

the equilibrium $\bar{x} = 1$ is not LAS.

3 γx_{n-1}

For the rational difference equation

$$x_{n+1} = \gamma x_{n-1}$$

the only non-negative equilibrium is $\bar{x} = 0$. If $\gamma \geq 1$ then $\bar{x}$ is not LAS. If $0 \leq \gamma < 1$ the $\bar{x}$ is LAS and GAS. The polynomial that must be positive is

$$P := x_{n-1}^2(1 - \gamma^2)$$

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and $0 \leq \gamma < 1$. Since there is a negative coefficient $(-\gamma^2)$ we do a quick substitution to get the range of γ to be $(0, \infty)$. First let $\gamma = \frac{1}{\gamma_1}$ and multiply by γ_1^2. Then $1 < \gamma_1 < \infty$. Next, let $\gamma_1 = \gamma_2 + 1$ so that $0 < \gamma_2 < \infty$. Now the polynomial is

$$x_{n-1}^2(\gamma_2^2 + 2\gamma_2)$$

which has all positive coefficients. Therefore, for $0 \leq \gamma < 1$ the equilibrium $\bar{x} = 0$ is GAS.

4 $\quad \dfrac{x_{n-1}}{x_n}$

For the rational difference equation

$$x_{n+1} = \frac{x_{n-1}}{x_n}$$

the equilibrium $\bar{x} = 1$ is not LAS.

5 $\quad \dfrac{1}{4}\dfrac{M^2-1}{1+x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - 1}{1 + x_{n-1}}$$

Conjecture 5.1. *For the equilibrium $\bar{x} = \frac{1}{2}(M-1)$, the following K values are conjectured depending on the value of M.*

- *If $1 \leq M \leq 3$ then $K = 2$.*

- *If $3 < M < \infty$ then $K = 4$.*

7 $\quad \dfrac{\beta x_n}{1+x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{\beta x_n}{1 + x_{n-1}}$$

there are two possible equilibrium points, $\bar{x} = 0$ and $\bar{x} = \beta - 1$. If $1 < \beta$ the equilibrium $\bar{x} = 0$ is not LAS, and we have no conjectures about the K value for the equilibrium $\bar{x} = \beta - 1$ (though we know it is LAS). If $0 < \beta < 1$ the only positive equilibrium is $\bar{x} = 0$ and it is LAS.

Conjecture 7.1. *For the equilibrium $\bar{x} = 0$ and in the range $0 < \beta < 1$, the following K values are conjectured depending on the value of β.*

- *If $0 \leq \beta \leq \frac{78}{100}$ then $K = 2$.*

- *If $\frac{79}{100} < \beta < \frac{90}{100}$ then $K = 4$.*

8 $\frac{x_n}{Bx_n+x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{x_n}{Bx_n + x_{n-1}}$$

the equilibrium is $\bar{x} = \frac{1}{B+1}$ and it is LAS for all $B > 0$.

Conjecture 8.1. *For the equilibrium $\bar{x} = \frac{1}{B+1}$, the following K values are conjectured depending on the value of B.*

- *If $\frac{11}{10} < B$ then $K = 3$.*

- *If $\frac{2}{5} < B < \frac{11}{10}$ then $K = 5$.*

- *If $\frac{1}{5} < B < \frac{2}{5}$ then $K = 6$.*

9 $\frac{x_{n-1}}{A+x_n}$

For the rational difference equation

$$x_{n+1} = \frac{x_{n-1}}{A + x_n}$$

there are two equilibrium points, $\bar{x} = 0$ and $\bar{x} = 1 - A$. If $A > 1$ the only non-negative equilibrium is $\bar{x} = 0$. In this case, $\bar{x}$ is LAS. To prove that it is GAS we must show that the following polynomial ($K = 1$) is positive,

$$P = x_n^2 x_{n-1}^2 + 2Ax_n x_{n-1}^2 + (A^2 - 1)x_{n-1}^2$$

when $A > 1$. So we make the substitution $A = A_1 + 1$, then the polynomial is

$$x_n^2 x_{n-1}^2 + (2A_1 + 2)x_n x_{n-1}^2 + (A_1^2 + 2A_1)x_{n-1}^2.$$

We are interested in the range $A_1 > 0$, and since all coefficients are positive, the polynomial is positive. Therefore $\bar{x} = 0$ is GAS when $A > 1$.

If $0 \le A < 1$ neither equilibrium is LAS.

10 $\frac{x_{n-1}}{A+x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{x_{n-1}}{A + x_{n-1}}$$

there are two possible equilibrium points, $\bar{x} = 0$ and $\bar{x} = 1 - A$. If $A > 1$ then the only non-negative equilibrium is $\bar{x} = 0$, and it is LAS. To prove that it is also GAS we must show that the following polynomial ($K = 1$) is positive for $A > 1$,

$$P := x_{n-1}^4 + 2Ax_{n-1}^3 + (A^2 - 1)x_{n-1}^2.$$

We do the substitution $A = A_1 + 1$ to get the polynomial

$$x_{n-1}^4 + (2A_1 + 2)x_{n-1}^3 + (A_1^2 + 2A_1)x_{n-1}^2.$$

Since the only variable in this polynomial is x_{n-1} (A_1 is a "parameter") we need to be careful. It could be the case that the polynomial has a zero when $x_{n-1} > 0$. If this happens then there are infinitely many zeros when $x_{n-1} > 0$ (the specific zero for x_{n-1}, paired with all values of x_n). Clearly $x_{n-1} = 0$ is a zero of the polynomial, but that is alright. Fortunately, the only other zeroes are negative, at $-A_1$ and $-A_1 - 2$. Therefore, the polynomial is strictly positive in the range $A_1 > 0$ as needed. Therefore $\bar{x} = 0$ is GAS when $A > 1$.

If $0 < A < 1$ then the equilibrium $\bar{x} = 0$ is not LAS. The equilibrium $\bar{x} = 1 - A$ is LAS.
Finish this one!

11 $\quad \dfrac{x_{n-1}}{Bx_n + x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{x_{n-1}}{Bx_n + x_{n-1}}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{B = 5\}, 1/6], [\{B = 24/5\}, 5/29], [\{B = 27/10\}, 10/37], [\{B = 29/20\}, 20/49], [\{B = 31/25\}, 25/56], [\{B = 37/10\}, 10/47], [\{B = 39/10\}, 10/49], [\{B = 39/20\}, 20/59], [\{B = 46/25\}, 25/71], [\{B = 48/5\}, 5/53], [\{B = 57/10\}, 10/67], [\{B = 61/20\}, 20/81], [\{B = 83/50\}, 50/133], [\{B = 99/20\}, 20/119], [\{B = 103/100\}, 100/203], [\{B = 117/25\}, 25/142], [\{B = 129/50\}, 50/179], [\{B = 131/25\}, 25/156], [\{B = 137/20\}, 20/157], [\{B = 147/50\}, 50/197], [\{B = 179/25\}, 25/204], [\{B = 183/25\}, 25/208], [\{B = 193/25\}, 25/218], [\{B = 263/50\}, 50/313], [\{B = 313/100\}, 100/413], [\{B = 317/100\}, 100/417], [\{B = 319/50\}, 50/369], [\{B = 321/50\}, 50/371], [\{B = 359/100\}, 100/459], [\{B = 361/50\}, 50/411], [\{B = 363/100\}, 100/463], [\{B = 371/50\}, 50/421], [\{B = 383/100\}, 100/483], [\{B = 389/50\}, 50/439], [\{B = 417/100\}, 100/517], [\{B = 421/100\}, 100/521], [\{B = 487/50\}, 50/537], [\{B = 497/50\}, 50/547], [\{B = 547/100\}, 100/647], [\{B = 601/100\}, 100/701], [\{B = 619/100\}, 100/719], [\{B = 631/100\}, 100/731], [\{B = 669/100\}, 100/769], [\{B = 683/100\}, 100/783], [\{B = 789/100\}, 100/889], [\{B = 807/100\}, 100/907],$

The parameter values and equilibrium for which the MaxK value, 6, , is not high enough are:

$$[\{B = 1/20\}, 20/21], [\{B = 23/100\}, 100/123], [\{B = 29/100\}, 100/129], [\{B = 47/100\}, 100/147],$$

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12 $\quad \frac{1}{4}\frac{M^2-1+4x_n}{x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - 1 + 4x_n}{x_{n-1}}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$$[\{M = 4\}, 5/2], [\{M = 5\}, 3], [\{M = 9/4\}, 13/8], [\{M = 14/5\}, 19/10], [\{M = 16/5\}, 21/10], [\{M = 27/10\}, 37/20], [\{M = 31/10\}, 41/20], [\{M = 87/50\}, 137/100], [\{M = 91/50\}, 141/100], [\{M = 123/20\}, 143/40], [\{M = 141/50\}, 191/100], [\{M = 148/25\}, 173/50], [\{M = 157/20\}, 177/40], [\{M = 174/25\}, 199/50], [\{M = 187/50\}, 237/100], [\{M = 191/25\}, 108/25], [\{M = 192/25\}, 217/50], [\{M = 193/20\}, 213/40], [\{M = 197/50\}, 247/100], [\{M = 203/100\}, 303/200], [\{M = 228/25\}, 253/50], [\{M = 233/25\}, 129/25], [\{M = 241/25\}, 133/25], [\{M = 244/25\}, 269/50], [\{M = 247/25\}, 136/25], [\{M = 251/100\}, 351/200], [\{M = 277/100\}, 377/200], [\{M = 283/50\}, 333/100], [\{M = 291/50\}, 341/100], [\{M = 299/50\}, 349/100], [\{M = 299/100\}, 399/200], [\{M = 353/100\}, 453/200], [\{M = 441/50\}, 491/100], [\{M = 441/100\}, 541/200], [\{M = 473/50\}, 523/100], [\{M = 493/100\}, 593/200], [\{M = 609/100\}, 709/200], [\{M = 641/100\}, 741/200], [\{M = 707/100\}, 807/200], [\{M = 747/100\}, 847/200], [\{M = 757/100\}, 857/200], [\{M = 787/100\}, 887/200], [\{M = 801/100\}, 901/200], [\{M = 859/100\}, 959/200], [\{M = 881/100\}, 981/200], [\{M = 901/100\}, 1001/200], [\{M = 933/100\}, 1033/200], [\{M = 961/100\}, 1061/200], [\{M = 963/100\}, 1063/200], [\{M = 973/100\}, 1073/200],$$

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13 $\quad \alpha + gx_{n-1}$

For the rational difference equation

$$x_{n+1} = \alpha + gx_{n-1}$$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{\alpha = 9, g = 3/5\}, 45/2], [\{\alpha = 1/50, g = 21/50\}, 1/29], [\{\alpha = 4/5, g = 13/20\}, 16/7], [\{\alpha = 8/25, g = 13/20\}, 32/35], [\{\alpha = 9/5, g = 77/100\}, 180/23], [\{\alpha = 9/100, g = 43/50\}, 9/14], [\{\alpha = 13/10, g = 53/100\}, 130/47], [\{\alpha = 17/25, g = 47/100\}, 68/53], [\{\alpha = 18/25, g = 16/25\}, 2], [\{\alpha = 33/50, g = 12/25\}, 33/26], [\{\alpha = 37/50, g = 24/25\}, 37/2], [\{\alpha = 39/10, g = 83/100\}, 390/17], [\{\alpha = 46/5, g = 13/50\}, 460/37], [\{\alpha = 51/50, g = 43/100\}, 34/19], [\{\alpha = 59/100, g = 9/10\}, 59/10], [\{\alpha = 64/25, g = 18/25\}, 64/7], [\{\alpha = 99/100, g = 1/5\}, 99/80], [\{\alpha = 103/25, g = 21/50\}, 206/29], [\{\alpha = 128/25, g = 77/100\}, 512/23], [\{\alpha = 147/25, g = 19/20\}, 588/5], [\{\alpha = 151/20, g = 24/25\}, 755/4], [\{\alpha = 156/25, g = 61/100\}, 16], [\{\alpha = 158/25, g = 24/25\}, 158], [\{\alpha = 161/100, g = 13/50\}, 161/74], [\{\alpha = 169/100, g = 41/50\}, 169/18], [\{\alpha = 173/100, g = 3/5\}, 173/40], [\{\alpha = 183/25, g = 3/20\}, 732/85], [\{\alpha = 189/100, g = 63/100\}, 189/37], [\{\alpha = 199/50, g = 17/50\}, 199/33], [\{\alpha = 203/50, g = 19/20\}, 406/5], [\{\alpha = 208/25, g = 7/100\}, 832/93], [\{\alpha = 213/100, g = 1/50\}, 213/98], [\{\alpha = 218/25, g = 1/10\}, 436/45], [\{\alpha = 242/25, g = 49/50\}, 484], [\{\alpha = 247/25, g = 1/5\}, 247/20], [\{\alpha = 259/100, g = 73/100\}, 259/27], [\{\alpha = 331/50, g = 14/25\}, 331/22], [\{\alpha = 381/50, g = 27/50\}, 381/23], [\{\alpha = 419/100, g = 3/50\}, 419/94], [\{\alpha = 443/50, g = 21/25\}, 443/8], [\{\alpha = 459/50, g = 13/50\}, 459/37], [\{\alpha = 461/50, g = 51/100\}, 922/49], [\{\alpha = 493/50, g = 1/5\}, 493/40], [\{\alpha = 499/100, g = 1/50\}, 499/98], [\{\alpha = 619/100, g = 7/100\}, 619/93], [\{\alpha = 791/100, g = 13/100\}, 791/87], [\{\alpha = 837/100, g = 7/10\}, 279/10], [\{\alpha = 881/100, g = 1/100\}, 881/99], [\{\alpha = 889/100, g = 53/100\}, 889/47], [\{\alpha = 983/100, g = 83/100\}, 983/17],$

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14 $\dfrac{1}{4}\dfrac{M^2-1+4x_{n-1}}{x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - 1 + 4x_{n-1}}{x_{n-1}}$$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 9/5\}, 7/5], [\{M = 13/4\}, 17/8], [\{M = 18/5\}, 23/10], [\{M = 23/4\}, 27/8], [\{M = 29/20\}, 49/40], [\{M = 32/5\}, 37/10], [\{M = 33/10\}, 43/20], [\{M = 41/20\}, 61/40], [\{M = 43/25\}, 34/25], [\{M = 51/50\}, 101/100], [\{M = 53/25\}, 39/25], [\{M = 68/25\}, 93/50], [\{M = 69/20\}, 89/40], [\{M = 73/25\}, 49/25], [\{M = 87/50\}, 137/100], [\{M = 98/25\}, 123/50], [\{M = 99/20\}, 119/40], [\{M = 106/25\}, 131/50], [\{M = 108/25\}, 133/50], [\{M = 111/100\}, 211/200], [\{M = 123/25\}, 74/25], [\{M = 141/50\}, 191/100], [\{M = 159/20\}, 179/40], [\{M =$

$167/25\}, 96/25], [\{M = 168/25\}, 193/50], [\{M = 171/100\}, 271/200], [\{M = 177/100\}, 277/200], [\{M = 189/100\}, 289/200], [\{M = 194/25\}, 219/50], [\{M = 198/25\}, 223/50], [\{M = 201/100\}, 301/200], [\{M = 217/25\}, 121/25], [\{M = 219/50\}, 269/100], [\{M = 227/100\}, 327/200], [\{M = 261/50\}, 311/100], [\{M = 289/100\}, 389/200], [\{M = 293/50\}, 343/100], [\{M = 293/100\}, 393/200], [\{M = 309/50\}, 359/100], [\{M = 341/100\}, 441/200], [\{M = 347/50\}, 397/100], [\{M = 357/50\}, 407/100], [\{M = 399/50\}, 449/100], [\{M = 399/100\}, 499/200], [\{M = 417/100\}, 517/200], [\{M = 461/50\}, 511/100], [\{M = 563/100\}, 663/200], [\{M = 639/100\}, 739/200], [\{M = 837/100\}, 937/200], [\{M = 937/100\}, 1037/200],$

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15 $\dfrac{1}{4}\dfrac{M^2-q^2+4qx_{n-1}}{x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 + 4qx_{n-1}}{x_{n-1}}$$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 39/4, q = 5/4\}, 11/2], [\{M = 39/5, q = 57/50\}, 447/100], [\{M = 58/25, q = 227/100\}, 459/200], [\{M = 79/20, q = 41/50\}, 477/200], [\{M = 83/10, q = 44/25\}, 503/100], [\{M = 83/10, q = 117/50\}, 133/25], [\{M = 93/50, q = 31/20\}, 341/200], [\{M = 129/20, q = 193/50\}, 1031/200], [\{M = 129/20, q = 267/100\}, 114/25], [\{M = 139/25, q = 127/25\}, 133/25], [\{M = 143/25, q = 517/100\}, 1089/200], [\{M = 157/20, q = 53/10\}, 263/40], [\{M = 158/25, q = 313/50\}, 629/100], [\{M = 159/20, q = 233/100\}, 257/50], [\{M = 164/25, q = 27/20\}, 791/200], [\{M = 169/25, q = 603/100\}, 1279/200], [\{M = 171/20, q = 57/20\}, 57/10], [\{M = 171/25, q = 98/25\}, 269/50], [\{M = 174/25, q = 47/20\}, 931/200], [\{M = 179/25, q = 117/25\}, 148/25], [\{M = 181/20, q = 12/5\}, 229/40], [\{M = 183/25, q = 253/50\}, 619/100], [\{M = 199/20, q = 242/25\}, 1963/200], [\{M = 199/20, q = 611/100\}, 803/100], [\{M = 217/25, q = 279/50\}, 713/100], [\{M = 218/25, q = 833/100\}, 341/40], [\{M = 223/100, q = 77/50\}, 377/200], [\{M = 227/25, q = 407/100\}, 263/40], [\{M = 243/25, q = 29/5\}, 194/25], [\{M = 243/25, q = 231/25\}, 237/25], [\{M = 243/100, q = 129/100\}, 93/50], [\{M = 257/50, q = 179/50\}, 109/25], [\{M = 271/50, q = 1/100\}, 543/200], [\{M = 291/50, q = 31/50\}, 161/50], [\{M = 293/100, q = 2\}, 493/200], [\{M = 299/100, q = 113/50\}, 21/8], [\{M = 321/50, q = 12/25\}, 69/20], [\{M = 321/50, q = 229/50\}, 11/2], [\{M = 343/50, q = 17/5\}, 513/100], [\{M = 401/50, q = 109/100\}, 911/200], [\{M = 427/100, q = 119/50\}, 133/40], [\{M = 473/50, q = $

$89/20\}, 1391/200], [\{M = 611/100, q = 1/100\}, 153/50], [\{M = 627/100, q = 201/50\}, 1029/200], [\{M = 839/100, q = 24/25\}, 187/40], [\{M = 867/100, q = 19/25\}, 943/200], [\{M = 869/100, q = 703/100\}, 393/50], [\{M = 871/100, q = 369/100\}, 31/5], [\{M = 901/100, q = 717/100\}, 809/100], [\{M = 903/100, q = 731/100\}, 817/100],$

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16 $\beta x_n + g x_{n-1}$

For the rational difference equation

$$x_{n+1} = \beta x_n + g x_{n-1}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{\beta = 13/25, g = 537/100\}, 0], [\{\beta = 17/50, g = 43/50\}, 0], [\{\beta = 19/50, g = 993/100\}, 0], [\{\beta = 23/4, g = 107/100\}, 0], [\{\beta = 24/25, g = 891/100\}, 0], [\{\beta = 27/10, g = 573/100\}, 0], [\{\beta = 31/50, g = 10\}, 0], [\{\beta = 33/4, g = 933/100\}, 0], [\{\beta = 33/25, g = 647/100\}, 0], [\{\beta = 34/25, g = 26/5\}, 0], [\{\beta = 47/25, g = 7/25\}, 0], [\{\beta = 51/10, g = 961/100\}, 0], [\{\beta = 51/25, g = 613/100\}, 0], [\{\beta = 91/100, g = 331/50\}, 0], [\{\beta = 93/10, g = 32/25\}, 0], [\{\beta = 124/25, g = 57/50\}, 0], [\{\beta = 142/25, g = 69/50\}, 0], [\{\beta = 149/20, g = 46/25\}, 0], [\{\beta = 149/25, g = 69/20\}, 0], [\{\beta = 161/50, g = 67/20\}, 0], [\{\beta = 163/20, g = 227/25\}, 0], [\{\beta = 179/25, g = 27/100\}, 0], [\{\beta = 187/50, g = 13/100\}, 0], [\{\beta = 191/25, g = 549/100\}, 0], [\{\beta = 223/100, g = 81/25\}, 0], [\{\beta = 239/25, g = 73/50\}, 0], [\{\beta = 247/100, g = 79/25\}, 0], [\{\beta = 247/100, g = 157/25\}, 0], [\{\beta = 267/50, g = 587/100\}, 0], [\{\beta = 267/100, g = 813/100\}, 0], [\{\beta = 271/50, g = 193/50\}, 0], [\{\beta = 303/50, g = 137/20\}, 0], [\{\beta = 317/50, g = 889/100\}, 0], [\{\beta = 357/100, g = 13/10\}, 0], [\{\beta = 371/100, g = 7/50\}, 0], [\{\beta = 387/50, g = 713/100\}, 0], [\{\beta = 451/50, g = 173/20\}, 0], [\{\beta = 489/100, g = 99/100\}, 0], [\{\beta = 503/100, g = 233/50\}, 0], [\{\beta = 507/100, g = 741/100\}, 0], [\{\beta = 531/100, g = 311/50\}, 0], [\{\beta = 589/100, g = 289/100\}, 0], [\{\beta = 631/100, g = 83/20\}, 0], [\{\beta = 699/100, g = 369/100\}, 0], [\{\beta = 699/100, g = 467/100\}, 0], [\{\beta = 721/100, g = 89/100\}, 0], [\{\beta = 757/100, g = 391/100\}, 0], [\{\beta = 897/100, g = 413/100\}, 0], [\{\beta = 937/100, g = 81/100\}, 0], [\{\beta = 949/100, g = 339/100\}, 0],$

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17 $\beta + \frac{x_{n-1}}{x_n}$

For the rational difference equation

$$x_{n+1} = \beta + \frac{x_{n-1}}{x_n}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$$[\{\beta = 7/25\}, 32/25], [\{\beta = 69/100\}, 169/100],$$

The parameter values and equilibrium for which the MaxK value, 6, , is not high enough are:

$$[\{\beta = 7\}, 8], [\{\beta = 13/5\}, 18/5], [\{\beta = 13/10\}, 23/10], [\{\beta = 23/10\}, 33/10], [\{\beta = 23/20\}, 43/20], [\{\beta = 61/25\}, 86/25], [\{\beta = 72/25\}, 97/25], [\{\beta = 86/25\}, 111/25], [\{\beta = 96/25\}, 121/25], [\{\beta = 129/50\}, 179/50], [\{\beta = 147/20\}, 167/20], [\{\beta = 161/25\}, 186/25], [\{\beta = 163/25\}, 188/25], [\{\beta = 166/25\}, 191/25], [\{\beta = 171/20\}, 191/20], [\{\beta = 197/50\}, 247/50], [\{\beta = 203/100\}, 303/100], [\{\beta = 211/25\}, 236/25], [\{\beta = 211/50\}, 261/50], [\{\beta = 217/50\}, 267/50], [\{\beta = 238/25\}, 263/25], [\{\beta = 239/50\}, 289/50], [\{\beta = 242/25\}, 267/25], [\{\beta = 253/100\}, 353/100], [\{\beta = 259/50\}, 309/50], [\{\beta = 261/100\}, 361/100], [\{\beta = 289/50\}, 339/50], [\{\beta = 311/50\}, 361/50], [\{\beta = 347/100\}, 447/100], [\{\beta = 363/50\}, 413/50], [\{\beta = 363/100\}, 463/100], [\{\beta = 391/50\}, 441/50], [\{\beta = 397/100\}, 497/100], [\{\beta = 519/100\}, 619/100], [\{\beta = 541/100\}, 641/100], [\{\beta = 549/100\}, 649/100], [\{\beta = 583/100\}, 683/100], [\{\beta = 687/100\}, 787/100], [\{\beta = 729/100\}, 829/100], [\{\beta = 769/100\}, 869/100], [\{\beta = 793/100\}, 893/100], [\{\beta = 861/100\}, 961/100], [\{\beta = 877/100\}, 977/100], [\{\beta = 899/100\}, 999/100], [\{\beta = 933/100\}, 1033/100], [\{\beta = 957/100\}, 1057/100], [\{\beta = 967/100\}, 1067/100], [\{\beta = 983/100\}, 1083/100],$$

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18 $g + \frac{x_n}{x_{n-1}}$

For the rational difference equation

$$x_{n+1} = g + \frac{x_n}{x_{n-1}}$$

The parameter values and equilibrium for which the MaxK value, 6, , is not high enough are:

$$[\{g = 3/50\}, 53/50], [\{g = 4/25\}, 29/25], [\{g = 11/50\}, 61/50], [\{g = 31/100\}, 131/100],$$

The parameter values and equilibrium for which the $K = 5$ are:

$[\{g = 17/25\}, 42/25], [\{g = 19/4\}, 23/4], [\{g = 21/10\}, 31/10], [\{g = 32/25\}, 57/25], [\{g = 33/5\}, 38/5], [\{g = 36/25\}, 61/25], [\{g = 39/25\}, 64/25], [\{g = 49/50\}, 99/50], [\{g = 59/50\}, 109/50], [\{g = 63/100\}, 163/100], [\{g = 87/100\}, 187/100], [\{g = 136/25\}, 161/25], [\{g = 143/50\}, 193/50], [\{g = 193/50\}, 243/50], [\{g = 199/50\}, 249/50], [\{g = 201/50\}, 251/50], [\{g = 212/25\}, 237/25], [\{g = 217/50\}, 267/50], [\{g = 219/25\}, 244/25], [\{g = 219/100\}, 319/100], [\{g = 223/25\}, 248/25], [\{g = 228/25\}, 253/25], [\{g = 229/25\}, 254/25], [\{g = 231/25\}, 256/25], [\{g = 233/100\}, 333/100], [\{g = 241/100\}, 341/100], [\{g = 273/100\}, 373/100], [\{g = 293/100\}, 393/100], [\{g = 303/50\}, 353/50], [\{g = 317/50\}, 367/50], [\{g = 397/100\}, 497/100], [\{g = 401/100\}, 501/100], [\{g = 453/100\}, 553/100], [\{g = 527/100\}, 627/100], [\{g = 573/100\}, 673/100], [\{g = 603/100\}, 703/100], [\{g = 639/100\}, 739/100], [\{g = 653/100\}, 753/100], [\{g = 723/100\}, 823/100], [\{g = 739/100\}, 839/100], [\{g = 767/100\}, 867/100], [\{g = 811/100\}, 911/100], [\{g = 861/100\}, 961/100], [\{g = 869/100\}, 969/100], [\{g = 901/100\}, 1001/100], [\{g = 963/100\}, 1063/100],$

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19 $\dfrac{1}{4}\dfrac{M^2 - q^2 + 4x_n}{1 + q + x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 + 4x_n}{1 + q + x_{n-1}}$$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 29/4, q = 57/25\}, 497/200], [\{M = 33/5, q = 289/100\}, 371/200], [\{M = 39/4, q = 228/25\}, 63/200], [\{M = 61/10, q = 237/50\}, 17/25], [\{M = 83/50, q = 137/100\}, 29/200], [\{M = 91/10, q = 137/25\}, 181/100], [\{M = 119/25, q = 44/25\}, 3/2], [\{M = 121/50, q = 111/100\}, 131/200], [\{M = 142/25, q = 421/100\}, 147/200], [\{M = 153/25, q = 173/100\}, 439/200], [\{M = 159/25, q = 297/50\}, 21/100], [\{M = 161/20, q = 7\}, 21/40], [\{M = 163/20, q = 108/25\}, 383/200], [\{M = 163/25, q = 601/100\}, 51/200], [\{M = 189/25, q = 637/100\}, 119/200], [\{M = 213/25, q = 124/25\}, 89/50], [\{M = 218/25, q = 112/25\}, 53/25], [\{M = 229/25, q = 451/50\}, 7/100], [\{M = 233/25, q = 273/100\}, 659/200], [\{M = 241/25, q = 57/10\}, 197/100], [\{M = 241/25, q = 249/50\}, 233/100], [\{M = 247/100, q = 59/25\}, 11/200], [\{M = 248/25, q = 172/25\}, 38/25], [\{M = 303/100, q = 283/100\}, 1/10], [\{M = 339/100, q =$

$79/50\}, 181/200], [\{M = 343/50, q = 93/20\}, 221/200], [\{M = 363/100, q = 42/25\}, 39/40], [\{M = 371/50, q = 52/25\}, 267/100], [\{M = 379/100, q = 271/100\}, 27/50], [\{M = 421/50, q = 337/50\}, 21/25], [\{M = 467/100, q = 123/100\}, 43/25], [\{M = 533/100, q = 151/50\}, 231/200], [\{M = 627/100, q = 169/100\}, 229/100], [\{M = 701/100, q = 74/25\}, 81/40], [\{M = 929/100, q = 17/5\}, 589/200], [\{M = 941/100, q = 319/100\}, 311/100], [\{M = 973/100, q = 637/100\}, 42/25], [\{M = 997/100, q = 573/100\}, 53/25],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{M = 91/10, q = 241/100\}, 669/200], [\{M = 187/20, q = 243/100\}, 173/50], [\{M = 841/100, q = 109/50\}, 623/200],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 199/50, q = 17/100\}, 381/200], [\{M = 313/50, q = 31/50\}, 141/50], [\{M = 541/100, q = 43/100\}, 249/100], [\{M = 603/100, q = 3/5\}, 543/200], [\{M = 799/100, q = 29/20\}, 327/100],$

The parameter values and equilibrium for which the $K = 5$ are:

$[\{M = 137/25, q = 9/100\}, 539/200], [\{M = 141/20, q = 7/25\}, 677/200], [\{M = 269/50, q = 4/25\}, 261/100], [\{M = 771/100, q = 17/50\}, 737/200],$

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21 $\dfrac{1}{4}\dfrac{M^2-q^2+4x_{n-1}}{1+q+x_n}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 + 4x_{n-1}}{1 + q + x_n}$$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{M = 19/20, q = 21/100\}, 37/100], [\{M = 44/5, q = 23/50\}, 417/100], [\{M = 117/20, q = 1/50\}, 583/200], [\{M = 377/100, q = 41/100\}, 42/25], [\{M = 413/100, q = 7/20\}, 189/100], [\{M = 417/100, q = 1/50\}, 83/40], [\{M = 743/100, q = 11/25\}, 699/200], [\{M = 803/100, q = 21/50\}, 761/200],$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 15/2, q = 279/50\}, 24/25], [\{M = 58/25, q = 23/20\}, 117/200], [\{M = 63/10, q = 463/100\}, 167/200], [\{M = 67/50, q = 117/100\}, 17/200], [\{M = 79/25, q = 207/100\}, 109/200], [\{M = 89/10, q = 513/100\}, 377/200], [\{M = 97/10, q = 237/50\}, 62/25], [\{M = 99/10, q = 713/100\}, 277/200], [\{M = 119/25, q = 21/5\}, 7/25], [\{M = 126/25, q = 183/50\}, 69/100], [\{M = 166/25, q = 193/100\}, 471/200], [\{M = 204/25, q = 68/25\}, 68/25], [\{M = 228/25, q = 138/25\}, 9/5], [\{M = 253/50, q = 327/100\}, 179/200], [\{M = 281/50, q = 64/25\}, 153/100], [\{M = 413/50, q = 24/5\}, 173/100], [\{M = 437/50, q = 173/20\}, 9/200], [\{M = 483/100, q = 109/50\}, 53/40], [\{M = 521/100, q = 92/25\}, 153/200], [\{M = 569/100, q = 113/20\}, 1/50], [\{M = 633/100, q = 69/25\}, 357/200], [\{M = 649/100, q = 259/100\}, 39/20], [\{M = 649/100, q = 311/50\}, 27/200], [\{M = 671/100, q = 119/20\}, 19/50], [\{M = 691/100, q = 183/50\}, 13/8], [\{M = 733/100, q = 701/100\}, 4/25], [\{M = 751/100, q = 693/100\}, 29/100], [\{M = 759/100, q = 703/100\}, 7/25], [\{M = 779/100, q = 181/50\}, 417/200], [\{M = 807/100, q = 191/50\}, 17/8], [\{M = 899/100, q = 757/100\}, 71/100], [\{M = 911/100, q = 301/100\}, 61/20], [\{M = 937/100, q = 163/25\}, 57/40], [\{M = 981/100, q = 373/100\}, 76/25],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{M = 35/4, q = 219/100\}, 82/25], [\{M = 124/25, q = 19/20\}, 401/200], [\{M = 219/100, q = 14/25\}, 163/200], [\{M = 889/100, q = 49/20\}, 161/50], [\{M = 937/100, q = 42/25\}, 769/200],$

The parameter values and equilibrium for which the $K = 5$ are:

$[\{M = 174/25, q = 17/20\}, 611/200], [\{M = 551/100, q = 37/50\}, 477/200], [\{M = 703/100, q = 81/100\}, 311/100],$

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22 $\quad \dfrac{1}{4}\dfrac{M^2-q^2+4x_{n-1}}{1+q+x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 + 4x_{n-1}}{1 + q + x_{n-1}}$$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 13/2, q = 19/4\}, 7/8], [\{M = 13/2, q = 197/100\}, 453/200], [\{M = 29/10, q = 137/100\}, 153/200], [\{M = 33/20, q = 33/100\}, 33/50], [\{M = 47/100, q =$

$3/10\}, 17/200], [\{M = 113/20, q = 307/100\}, 129/100], [\{M = 123/25, q = 159/100\}, 333/200], [\{M = 139/25, q = 11/4\}, 281/200], [\{M = 141/50, q = 63/50\}, 39/50], [\{M = 147/25, q = 299/100\}, 289/200], [\{M = 159/25, q = 249/50\}, 69/100], [\{M = 189/50, q = 359/100\}, 19/200], [\{M = 191/20, q = 203/50\}, 549/200], [\{M = 203/25, q = 151/25\}, 26/25], [\{M = 207/50, q = 12/25\}, 183/100], [\{M = 207/100, q = 9/5\}, 27/200], [\{M = 209/100, q = 121/100\}, 11/25], [\{M = 211/25, q = 19/5\}, 58/25], [\{M = 214/25, q = 54/25\}, 16/5], [\{M = 219/50, q = 19/100\}, 419/200], [\{M = 229/25, q = 91/25\}, 69/25], [\{M = 247/50, q = 17/25\}, 213/100], [\{M = 269/100, q = 4/5\}, 189/200], [\{M = 279/50, q = 73/25\}, 133/100], [\{M = 393/50, q = 79/25\}, 47/20], [\{M = 411/50, q = 383/50\}, 7/25], [\{M = 427/100, q = 68/25\}, 31/40], [\{M = 449/100, q = 19/25\}, 373/200], [\{M = 479/100, q = 77/25\}, 171/200], [\{M = 489/100, q = 369/100\}, 3/5], [\{M = 497/100, q = 73/25\}, 41/40], [\{M = 499/50, q = 79/25\}, 341/100], [\{M = 599/100, q = 139/100\}, 23/10], [\{M = 633/100, q = 341/100\}, 73/50], [\{M = 661/100, q = 103/100\}, 279/100], [\{M = 761/100, q = 47/20\}, 263/100], [\{M = 783/100, q = 3\}, 483/200], [\{M = 829/100, q = 739/100\}, 9/20], [\{M = 891/100, q = 653/100\}, 119/100], [\{M = 911/100, q = 843/100\}, 17/50], [\{M = 927/100, q = 441/100\}, 243/100], [\{M = 929/100, q = 687/100\}, 121/100], [\{M = 941/100, q = 271/100\}, 67/20], [\{M = 983/100, q = 877/100\}, 53/100],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 48/5, q = 123/100\}, 837/200], [\{M = 139/25, q = 2/25\}, 137/50], [\{M = 174/25, q = 59/100\}, 637/200], [\{M = 371/50, q = 53/50\}, 159/50], [\{M = 419/50, q = 1/25\}, 417/100], [\{M = 829/100, q = 41/50\}, 747/200],$

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23 $\quad \dfrac{1}{4}\dfrac{M^2q^2-q^2+4qx_{n-1}}{x_n(1-q)+qx_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2q^2 - q^2 + 4qx_{n-1}}{x_n(1-q) + qx_{n-1}}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{M = 151/100, q = 7/50\}, 1757/10000], [\{M = 196/25, q = 7/100\}, 1547/5000], [\{M = 281/100, q = 1/20\}, 381/4000], [\{M = 431/100, q = 9/50\}, 4779/10000], [\{M = 721/100, q = 3/25\}, 2463/5000],$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$$[\{M = 37/4, q = 33/50\}, 1353/400], [\{M = 81/10, q = 21/25\}, 1911/500], [\{M = 167/25, q = 81/100\}, 1944/625], [\{M = 191/25, q = 67/100\}, 1809/625], [\{M = 213/25, q = 79/100\}, 9401/2500], [\{M = 409/50, q = 89/100\}, 40851/10000], [\{M = 423/100, q = 99/100\}, 51777/20000], [\{M = 427/50, q = 7/10\}, 3339/1000], [\{M = 523/100, q = 3/4\}, 1869/800], [\{M = 747/100, q = 24/25\}, 2541/625], [\{M = 791/100, q = 67/100\}, 59697/20000], [\{M = 857/100, q = 11/100\}, 10527/20000], [\{M = 857/100, q = 41/50\}, 39237/10000], [\{M = 909/100, q = 11/100\}, 11099/20000],$$

The parameter values and equilibrium for which the $K = 3$ are:

$$[\{M = 10, q = 17/50\}, 187/100], [\{M = 7/4, q = 49/100\}, 539/800], [\{M = 11/4, q = 19/25\}, 57/40], [\{M = 63/20, q = 12/25\}, 249/250], [\{M = 71/10, q = 1/4\}, 81/80], [\{M = 109/25, q = 7/20\}, 469/500], [\{M = 111/20, q = 11/20\}, 1441/800], [\{M = 121/50, q = 43/100\}, 7353/10000], [\{M = 163/50, q = 19/50\}, 4047/5000], [\{M = 169/100, q = 63/100\}, 16947/20000], [\{M = 176/25, q = 27/100\}, 5427/5000], [\{M = 207/100, q = 51/100\}, 15657/20000], [\{M = 226/25, q = 57/100\}, 14307/5000], [\{M = 229/25, q = 39/100\}, 4953/2500], [\{M = 307/50, q = 9/20\}, 3213/2000], [\{M = 383/100, q = 43/100\}, 20769/20000], [\{M = 389/100, q = 37/100\}, 18093/20000], [\{M = 427/50, q = 2/5\}, 477/250], [\{M = 519/100, q = 61/100\}, 37759/20000], [\{M = 631/100, q = 29/50\}, 21199/10000], [\{M = 709/100, q = 11/25\}, 8899/5000], [\{M = 821/100, q = 61/100\}, 56181/20000], [\{M = 897/100, q = 49/100\}, 48853/20000], [\{M = 911/100, q = 23/50\}, 23253/10000], [\{M = 959/100, q = 2/5\}, 1059/500],$$

The parameter values and equilibrium for which the $K = 4$ are:

$$[\{M = 68/25, q = 9/25\}, 837/1250],$$

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24 $\quad \frac{x_n + gx_{n-1}}{A + x_n}$

For the rational difference equation

$$x_{n+1} = \frac{x_n + gx_{n-1}}{A + x_n}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{A = 4, g = 369/100\}, 0], [\{A = 1/25, g = 161/50\}, 0], [\{A = 1/25, g = 161/50\}, 209/50], [\{A = 1/50, g = 481/50\}, 0], [\{A = 1/50, g = 481/50\}, 53/5], [\{A = 7/5, g = 73/50\}, 0], [\{A = 7/100, g = 117/25\}, 0], [\{A = 7/100, g = 117/25\}, 561/100], [\{A = 12/5, g = 581/100\}, 0], [\{A = 12/5, g = 581/100\}, 441/100], [\{A = 13/10, g = 677/100\}, 0], [\{A = 13/10, g = 677/100\}, 647/100], [\{A = 13/20, g = 69/10\}, 0], [\{A = 13/20, g = 69/10\}, 29/4], [\{A = 19/50, g = 34/5\}, 0], [\{A = 19/50, g = 34/5\}, 371/50], [\{A = 22/5, g = 283/50\}, 0], [\{A = 22/5, g = 283/50\}, 113/50], [\{A = 33/25, g = 216/25\}, 0], [\{A = 33/25, g = 216/25\}, 208/25], [\{A = 37/50, g = 989/100\}, 0], [\{A = 37/50, g = 989/100\}, 203/20], [\{A = 43/20, g = 88/25\}, 0], [\{A = 43/20, g = 88/25\}, 237/100], [\{A = 43/50, g = 246/25\}, 0], [\{A = 43/50, g = 246/25\}, 499/50], [\{A = 43/100, g = 31/100\}, 0], [\{A = 58/25, g = 57/10\}, 0], [\{A = 58/25, g = 57/10\}, 219/50], [\{A = 62/25, g = 553/100\}, 0], [\{A = 62/25, g = 553/100\}, 81/20], [\{A = 63/20, g = 363/50\}, 0], [\{A = 63/20, g = 363/50\}, 511/100], [\{A = 63/100, g = 12/5\}, 0], [\{A = 63/100, g = 12/5\}, 277/100], [\{A = 63/100, g = 19/5\}, 0], [\{A = 63/100, g = 19/5\}, 417/100], [\{A = 67/50, g = 613/100\}, 0], [\{A = 67/50, g = 613/100\}, 579/100], [\{A = 69/50, g = 77/100\}, 0], [\{A = 69/100, g = 141/100\}, 0], [\{A = 73/100, g = 863/100\}, 0], [\{A = 73/100, g = 863/100\}, 89/10], [\{A = 103/25, g = 138/25\}, 0], [\{A = 103/25, g = 138/25\}, 12/5], [\{A = 113/50, g = 63/20\}, 0], [\{A = 117/50, g = 19/10\}, 0], [\{A = 119/20, g = 244/25\}, 0], [\{A = 119/20, g = 244/25\}, 481/100], [\{A = 139/25, g = 861/100\}, 0], [\{A = 139/25, g = 861/100\}, 81/20], [\{A = 219/100, g = 97/10\}, 0], [\{A = 219/100, g = 97/10\}, 851/100], [\{A = 239/50, g = 481/50\}, 0], [\{A = 239/50, g = 481/50\}, 146/25], [\{A = 347/50, g = 233/25\}, 0], [\{A = 347/50, g = 233/25\}, 169/50],$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{A = 11/2, g = 91/100\}, 0], [\{A = 16/5, g = 77/100\}, 0], [\{A = 19/2, g = 24/5\}, 0], [\{A = 34/5, g = 309/100\}, 0], [\{A = 69/100, g = 141/100\}, 43/25], [\{A = 79/50, g = 3/25\}, 0], [\{A = 113/50, g = 63/20\}, 189/100], [\{A = 199/25, g = 53/10\}, 0], [\{A = 211/100, g = 7/100\}, 0], [\{A = 242/25, g = 138/25\}, 0], [\{A = 591/100, g = 479/100\}, 0], [\{A = 599/100, g = 99/25\}, 0], [\{A = 603/100, g = 33/50\}, 0], [\{A = 619/100, g = 24/25\}, 0], [\{A = 641/100, g = 19/10\}, 0], [\{A = 673/100, g = 29/100\}, 0], [\{A = 837/100, g = 379/100\}, 0], [\{A = 873/100, g = 537/100\}, 0], [\{A = 877/100, g = 117/50\}, 0], [\{A = 889/100, g = 11/20\}, 0],$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{A = 4, g = 369/100\}, 69/100], [\{A = 43/100, g = 31/100\}, 22/25], [\{A = 117/50, g = 19/10\}, 14/25],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{A = 7/5, g = 73/50\}, 53/50], [\{A = 69/50, g = 77/100\}, 39/100],$

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25 $\frac{\beta x_n + x_{n-1}}{A + x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{\beta x_n + x_{n-1}}{A + x_{n-1}}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{A = 1/20, \beta = 28/25\}, 0], [\{A = 2/25, \beta = 507/100\}, 0], [\{A = 3/50, \beta = 69/10\}, 0], [\{A = 12/25, \beta = 73/100\}, 0], [\{A = 16/5, \beta = 132/25\}, 0], [\{A = 21/5, \beta = 403/100\}, 0], [\{A = 22/25, \beta = 4/5\}, 0], [\{A = 39/20, \beta = 37/10\}, 0], [\{A = 43/25, \beta = 162/25\}, 0], [\{A = 46/25, \beta = 183/25\}, 0], [\{A = 47/20, \beta = 831/100\}, 0], [\{A = 48/25, \beta = 301/100\}, 0], [\{A = 49/5, \beta = 222/25\}, 0], [\{A = 59/100, \beta = 9/10\}, 0], [\{A = 111/100, \beta = 813/100\}, 0], [\{A = 114/25, \beta = 347/50\}, 0], [\{A = 119/50, \beta = 867/100\}, 0], [\{A = 124/25, \beta = 763/100\}, 0], [\{A = 129/100, \beta = 363/50\}, 0], [\{A = 139/20, \beta = 771/100\}, 0], [\{A = 143/50, \beta = 99/10\}, 0], [\{A = 152/25, \beta = 169/20\}, 0], [\{A = 193/100, \beta = 921/100\}, 0], [\{A = 223/25, \beta = 247/25\}, 0], [\{A = 291/50, \beta = 903/100\}, 0], [\{A = 299/100, \beta = 457/50\}, 0], [\{A = 311/100, \beta = 161/50\}, 0], [\{A = 339/50, \beta = 218/25\}, 0], [\{A = 343/50, \beta = 161/20\}, 0], [\{A = 393/100, \beta = 983/100\}, 0], [\{A = 449/100, \beta = 857/100\}, 0], [\{A = 451/100, \beta = 233/25\}, 0], [\{A = 451/100, \beta = 601/100\}, 0], [\{A = 487/100, \beta = 471/50\}, 0], [\{A = 547/100, \beta = 31/4\}, 0], [\{A = 653/100, \beta = 309/50\}, 0],$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{A = 1/20, \beta = 28/25\}, 207/100], [\{A = 2/25, \beta = 507/100\}, 599/100], [\{A = 3/50, \beta = 69/10\}, 196/25], [\{A = 21/5, \beta = 403/100\}, 83/100], [\{A = 37/5, \beta = 161/50\}, 0], [\{A = 43/25, \beta = 162/25\}, 144/25], [\{A = 46/25, \beta = 183/25\}, 162/25], [\{A = 47/20, \beta = 831/100\}, 174/25], [\{A = 49/5, \beta = 222/25\}, 2/25], [\{A = 53/10, \beta = 11/25\}, 0], [\{A = 111/100, \beta = 813/100\}, 401/50], [\{A = 119/50, \beta = 867/100\}, 729/100], [\{A = 129/100, \beta = 363/50\}, 697/100], [\{A = 139/20, \beta = 771/100\}, 44/25], [\{A = 158/25, \beta = 191/100\}, 0], [\{A = 163/25, \beta = 461/100\}, 0], [\{A = 193/25, \beta = 167/25\}, 0], [\{A = 193/100, \beta = 921/100\}, 207/25], [\{A = 197/50, \beta = 12/25\}, 0], [\{A = 223/25, \beta = 152/25\}, 0], [\{A = 223/25, \beta = 247/25\}, 49/25], [\{A = 247/25, \beta = 403/50\}, 0], [\{A = 339/50, \beta = 218/25\}, 147/50], [\{A = 343/50, \beta = 161/20\}, 219/100], [\{A = 367/50, \beta = 26/25\}, 0], [\{A = 403/100, \beta = 3/25\}, 0], [\{A = 653/100, \beta = 309/50\}, 13/20], [\{A = 853/100, \beta = 721/100\}, 0], [\{A = 871/100, \beta = 211/100\}, 0], [\{A = 891/100, \beta = 663/100\}, 0], [\{A = 979/100, \beta = 123/50\}, 0],$

The parameter values and equilibrium for which the $K = 2$ are:

$$[\{A = 22/25, \beta = 4/5\}, 23/25],$$

The parameter values and equilibrium for which the $K = 3$ are:

$$[\{A = 12/25, \beta = 73/100\}, 5/4], [\{A = 16/5, \beta = 132/25\}, 77/25], [\{A = 39/20, \beta = 37/10\}, 11/4], [\{A = 48/25, \beta = 301/100\}, 209/100], [\{A = 59/100, \beta = 9/10\}, 131/100],$$

The parameter values and equilibrium for which the $K = 4$ are:

$$[\{A = 114/25, \beta = 347/50\}, 169/50], [\{A = 124/25, \beta = 763/100\}, 367/100], [\{A = 143/50, \beta = 99/10\}, 201/25], [\{A = 299/100, \beta = 457/50\}, 143/20], [\{A = 393/100, \beta = 983/100\}, 69/10], [\{A = 449/100, \beta = 857/100\}, 127/25], [\{A = 451/100, \beta = 233/25\}, 581/100], [\{A = 487/100, \beta = 471/50\}, 111/20],$$

The parameter values and equilibrium for which the $K = 5$ are:

$$[\{A = 152/25, \beta = 169/20\}, 337/100], [\{A = 291/50, \beta = 903/100\}, 421/100], [\{A = 311/100, \beta = 161/50\}, 111/100], [\{A = 451/100, \beta = 601/100\}, 5/2], [\{A = 547/100, \beta = 31/4\}, 82/25],$$

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26 $\frac{\beta x_n + x_{n-1}}{B x_n + x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{\beta x_n + x_{n-1}}{B x_n + x_{n-1}}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$$[\{B = 19/2, \beta = 1/50\}, 17/175], [\{B = 343/50, \beta = 31/100\}, 1/6],$$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$$[\{B = 57/10, \beta = 61/100\}, 161/670], [\{B = 461/50, \beta = 79/100\}, 179/1022],$$

The parameter values and equilibrium for which the $K = 2$ are:

$$[\{B = 8/5, \beta = 133/50\}, 183/130], [\{B = 32/25, \beta = 172/25\}, 197/57], [\{B = 43/20, \beta = 327/100\}, 61/45], [\{B = 59/25, \beta = 367/50\}, 139/56], [\{B = 82/25, \beta = 244/25\}, 269/107], [\{B = 88/25, \beta = 241/25\}, 266/113], [\{B = 103/25, \beta = 887/100\}, 987/512], [\{B = 117/100, \beta = 94/25\}, 68/31], [\{B = 122/25, \beta = 148/25\}, 173/147], [\{B = 129/50, \beta = 71/50\}, 121/179], [\{B = 131/100, \beta = 433/100\}, 533/231], [\{B = 137/50, \beta = 203/100\}, 303/374], [\{B = 141/100, \beta =$$

$189/100\}, 289/241], [\{B = 143/25, \beta = 967/100\}, 1067/672], [\{B = 143/25, \beta =$
$971/100\}, 51/32], [\{B = 189/50, \beta = 871/100\}, 971/478], [\{B = 199/25, \beta =$
$413/100\}, 513/896], [\{B = 201/50, \beta = 87/20\}, 535/502], [\{B = 218/25, \beta =$
$499/100\}, 599/972], [\{B = 251/100, \beta = 214/25\}, 956/351], [\{B = 269/50, \beta =$
$311/50\}, 361/319], [\{B = 291/100, \beta = 18/5\}, 20/17], [\{B = 359/50, \beta =$
$77/10\}, 435/409], [\{B = 381/100, \beta = 74/25\}, 396/481], [\{B = 433/100, \beta =$
$253/50\}, 606/533], [\{B = 471/100, \beta = 923/100\}, 1023/571], [\{B = 477/100, \beta =$
$467/100\}, 567/577], [\{B = 627/100, \beta = 109/25\}, 536/727], [\{B = 661/100, \beta =$
$463/50\}, 1026/761], [\{B = 671/100, \beta = 249/50\}, 598/771], [\{B = 713/100, \beta =$
$453/100\}, 553/813], [\{B = 837/100, \beta = 109/25\}, 536/937], [\{B = 917/100, \beta =$
$319/50\}, 82/113],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{B = 13/2, \beta = 197/100\}, 99/250], [\{B = 93/100, \beta = 271/50\}, 642/193], [\{B =$
$331/50, \beta = 153/100\}, 253/762], [\{B = 479/50, \beta = 46/25\}, 142/529], [\{B = 811/100, \beta =$
$29/20\}, 245/911], [\{B = 839/100, \beta = 201/100\}, 301/939], [\{B = 901/100, \beta =$
$29/10\}, 30/77],$

The parameter values and equilibrium for which the $K = 5$ are:

$[\{B = 1/100, \beta = 61/25\}, 344/101], [\{B = 9/25, \beta = 289/100\}, 389/136], [\{B = 17/100, \beta =$
$177/25\}, 808/117], [\{B = 31/100, \beta = 78/25\}, 412/131], [\{B = 147/20, \beta =$
$9/10\}, 38/167], [\{B = 839/100, \beta = 59/50\}, 218/939],$

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27 $\dfrac{2M}{4M^2q-1+2Mx_n(q-1)+x_{n-1}2M}$

For the rational difference equation

$$x_{n+1} = \frac{2M}{4M^2q - 1 + 2Mx_n(q - 1) + x_{n-1}2M}$$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 3/5, q = 391/50\}, 125/1173], [\{M = 7/4, q = 299/100\}, 200/2093], [\{M = 13/2, q =$
$189/25\}, 25/2457], [\{M = 17/5, q = 397/100\}, 250/6749], [\{M = 19/10, q =$
$283/100\}, 500/5377], [\{M = 33/4, q = 333/50\}, 100/10989], [\{M = 42/25, q =$
$251/100\}, 625/5271], [\{M = 43/50, q = 247/100\}, 2500/10621], [\{M = 47/5, q =$
$763/100\}, 250/35861], [\{M = 47/20, q = 111/25\}, 250/5217], [\{M = 67/20, q =$
$721/100\}, 1000/48307], [\{M = 73/50, q = 271/50\}, 1250/19783], [\{M = 94/25, q =$

$227/50\}, 625/21338], [\{M = 99/10, q = 867/100\}, 500/85833], [\{M = 99/100, q = 269/50\}, 2500/26631], [\{M = 109/25, q = 137/100\}, 1250/14933], [\{M = 121/50, q = 963/100\}, 2500/116523], [\{M = 128/25, q = 247/50\}, 625/31616], [\{M = 139/20, q = 33/20\}, 200/4587], [\{M = 146/25, q = 513/100\}, 625/37449], [\{M = 148/25, q = 299/100\}, 625/22126], [\{M = 149/25, q = 439/50\}, 625/65411], [\{M = 177/50, q = 781/100\}, 2500/138237], [\{M = 227/100, q = 44/25\}, 625/4994], [\{M = 229/100, q = 959/100\}, 5000/219611], [\{M = 232/25, q = 147/25\}, 625/68208], [\{M = 239/25, q = 229/50\}, 625/54731], [\{M = 243/100, q = 353/100\}, 5000/85779], [\{M = 253/50, q = 113/20\}, 500/28589], [\{M = 311/50, q = 489/100\}, 2500/152079], [\{M = 313/50, q = 389/50\}, 1250/121757], [\{M = 343/100, q = 81/20\}, 1000/27783], [\{M = 369/100, q = 863/100\}, 5000/318447], [\{M = 453/100, q = 106/25\}, 625/24009], [\{M = 467/100, q = 73/50\}, 2500/34091], [\{M = 477/100, q = 217/25\}, 1250/103509], [\{M = 483/50, q = 32/25\}, 625/15456], [\{M = 493/50, q = 221/25\}, 625/108953], [\{M = 549/100, q = 89/25\}, 1250/48861], [\{M = 583/100, q = 371/100\}, 5000/216293], [\{M = 631/100, q = 399/100\}, 5000/251769], [\{M = 647/100, q = 33/4\}, 200/21351], [\{M = 651/100, q = 192/25\}, 625/62496], [\{M = 703/100, q = 78/25\}, 625/27417], [\{M = 709/100, q = 29/10\}, 500/20561], [\{M = 839/100, q = 757/100\}, 5000/635123], [\{M = 851/100, q = 213/25\}, 1250/181263], [\{M = 933/100, q = 69/50\}, 2500/64377], [\{M = 933/100, q = 187/25\}, 1250/174471], [\{M = 999/100, q = 459/50\}, 2500/458541],$

Finished investigating difference equation 27 out of 39

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28 $\frac{\beta x_n}{1+Bx_n+x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{\beta x_n}{1 + Bx_n + x_{n-1}}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{B = 3/5, \beta = 44/5\}, 0], [\{B = 9/5, \beta = 62/25\}, 0], [\{B = 17/20, \beta = 913/100\}, 0], [\{B = 21/5, \beta = 189/25\}, 0], [\{B = 26/5, \beta = 383/100\}, 0], [\{B = 34/25, \beta = 139/50\}, 0], [\{B = 44/5, \beta = 126/25\}, 0], [\{B = 61/100, \beta = 941/100\}, 0], [\{B = 79/25, \beta = 371/100\}, 0], [\{B = 83/25, \beta = 273/100\}, 0], [\{B = 88/25, \beta = 391/50\}, 0], [\{B = 129/50, \beta = 961/100\}, 0], [\{B = 129/100, \beta = 59/25\}, 0], [\{B = 149/20, \beta = 273/50\}, 0], [\{B = 151/20, \beta = 89/20\}, 0], [\{B = 201/100, \beta = 329/50\}, 0], [\{B = 201/100, \beta = 611/100\}, 0], [\{B = 227/50, \beta = 369/50\}, 0], [\{B = 238/25, \beta = 29/5\}, 0], [\{B = 239/50, \beta = 181/20\}, 0], [\{B = 247/25, \beta = 417/50\}, 0], [\{B = 289/100, \beta = 149/50\}, 0], [\{B = 331/50, \beta = 973/100\}, 0], [\{B = 333/100, \beta = 108/25\}, 0], [\{B = 339/50, \beta = 91/20\}, 0], [\{B = 351/100, \beta = 136/25\}, 0], [\{B = 353/100, \beta = 221/100\}, 0], [\{B =$

$357/100, \beta = 47/10\}, 0], [\{B = 361/50, \beta = 119/25\}, 0], [\{B = 367/100, \beta = 617/100\}, 0], [\{B = 391/100, \beta = 51/25\}, 0], [\{B = 451/50, \beta = 63/10\}, 0], [\{B = 499/100, \beta = 943/100\}, 0], [\{B = 617/100, \beta = 223/25\}, 0], [\{B = 807/100, \beta = 433/50\}, 0], [\{B = 901/100, \beta = 73/20\}, 0], [\{B = 923/100, \beta = 997/100\}, 0], [\{B = 953/100, \beta = 43/5\}, 0], [\{B = 973/100, \beta = 7/5\}, 0], [\{B = 987/100, \beta = 31/10\}, 0], [\{B = 987/100, \beta = 247/25\}, 0],$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{B = 3/5, \beta = 44/5\}, 39/8], [\{B = 9/5, \beta = 62/25\}, 37/70], [\{B = 17/20, \beta = 913/100\}, 813/185], [\{B = 21/5, \beta = 189/25\}, 82/65], [\{B = 26/5, \beta = 383/100\}, 283/620], [\{B = 34/25, \beta = 139/50\}, 89/118], [\{B = 44/5, \beta = 126/25\}, 101/245], [\{B = 61/100, \beta = 941/100\}, 841/161], [\{B = 79/25, \beta = 371/100\}, 271/416], [\{B = 83/25, \beta = 273/100\}, 173/432], [\{B = 88/25, \beta = 391/50\}, 341/226], [\{B = 129/50, \beta = 961/100\}, 861/358], [\{B = 129/100, \beta = 59/25\}, 136/229], [\{B = 149/20, \beta = 273/50\}, 446/845], [\{B = 151/20, \beta = 89/20\}, 23/57], [\{B = 201/100, \beta = 329/50\}, 558/301], [\{B = 201/100, \beta = 611/100\}, 73/43], [\{B = 227/50, \beta = 369/50\}, 319/277], [\{B = 238/25, \beta = 29/5\}, 120/263], [\{B = 239/50, \beta = 181/20\}, 805/578], [\{B = 247/25, \beta = 417/50\}, 367/544], [\{B = 289/100, \beta = 149/50\}, 198/389], [\{B = 331/50, \beta = 973/100\}, 291/254], [\{B = 333/100, \beta = 108/25\}, 332/433], [\{B = 339/50, \beta = 91/20\}, 355/778], [\{B = 351/100, \beta = 136/25\}, 444/451], [\{B = 353/100, \beta = 221/100\}, 121/453], [\{B = 357/100, \beta = 47/10\}, 370/457], [\{B = 361/50, \beta = 119/25\}, 188/411], [\{B = 367/100, \beta = 617/100\}, 517/467], [\{B = 391/100, \beta = 51/25\}, 104/491], [\{B = 451/50, \beta = 63/10\}, 265/501], [\{B = 499/100, \beta = 943/100\}, 843/599], [\{B = 617/100, \beta = 223/25\}, 264/239], [\{B = 807/100, \beta = 433/50\}, 766/907], [\{B = 901/100, \beta = 73/20\}, 265/1001], [\{B = 923/100, \beta = 997/100\}, 299/341], [\{B = 953/100, \beta = 43/5\}, 760/1053], [\{B = 973/100, \beta = 7/5\}, 40/1073], [\{B = 987/100, \beta = 31/10\}, 210/1087], [\{B = 987/100, \beta = 247/25\}, 888/1087],$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{B = 119/25, \beta = 41/100\}, 0], [\{B = 151/50, \beta = 13/50\}, 0], [\{B = 171/100, \beta = 1/10\}, 0], [\{B = 177/100, \beta = 47/100\}, 0], [\{B = 236/25, \beta = 39/100\}, 0], [\{B = 473/100, \beta = 3/20\}, 0], [\{B = 479/100, \beta = 2/25\}, 0], [\{B = 487/50, \beta = 3/25\}, 0], [\{B = 981/100, \beta = 67/100\}, 0],$

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29 $\frac{x_{n-1}}{A+Bx_n+x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{x_{n-1}}{A + Bx_n + x_{n-1}}$$

there are two possible equilibrium points, 0 and $\frac{1-A}{B+1}$. If $A > 1$ the only positive equilibrium is $\bar{x} = 0$, and it is LAS. In order to show that it is GAS we must prove that the following polynomial (for $K = 1$) is positive

$$P := x_{n-1}^4 + 2Bx_{n-1}^3 x_n + B^2 x_{n-1}^2 x_n^2 + 2Ax_{n-1}^3 + 2ABx_{n-1}^2 x_n + (A^2 - 1)x_{n-1}^2$$

when $A > 1$ and $B, x_n, x_{n-1} > 0$. In order to get all variables in the range $(0, \infty)$ we do the substitution $A = A_1 + 1$ and get

$$x_{n-1}^4 + 2Bx_{n-1}^3 x_n + B^2 x_{n-1}^2 x_n^2 + (2A_1 + 2)x_{n-1}^3 + (2BA_1 + 2B)x_{n-1}^2 x_n + (A_1^2 + 2A_1)x_{n-1}^2.$$

Since all coefficients are positive, the polynomial is positive when all variables and parameters are in the range $(0, \infty)$. Therefore, the equilibrium $\bar{x} = 0$ is GAS when $A > 1$.

If $0 < A < 1$ both equilibriums are not LAS.

30 $1 + \beta x_n + g x_{n-1}$

For the rational difference equation

$$x_{n+1} = 1 + \beta x_n + g x_{n-1}$$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$$[\{\beta = 13/20, g = 8/25\}, 100/3], [\{\beta = 18/25, g = 27/100\}, 100],$$

The parameter values and equilibrium for which the $K = 2$ are:

$$[\{\beta = 1/50, g = 11/100\}, 100/87], [\{\beta = 1/100, g = 61/100\}, 50/19], [\{\beta = 2/5, g = 1/100\}, 100/59], [\{\beta = 2/25, g = 4/25\}, 25/19], [\{\beta = 3/25, g = 23/100\}, 20/13], [\{\beta = 3/25, g = 37/50\}, 50/7], [\{\beta = 3/100, g = 8/25\}, 20/13], [\{\beta = 3/100, g = 21/100\}, 25/19], [\{\beta = 6/25, g = 1/50\}, 50/37], [\{\beta = 6/25, g = 3/50\}, 10/7], [\{\beta = 6/25, g = 61/100\}, 20/3], [\{\beta = 7/25, g = 16/25\}, 25/2], [\{\beta = 7/100, g = 7/100\}, 50/43], [\{\beta = 8/25, g = 2/5\}, 25/7], [\{\beta = 8/25, g = 3/50\}, 50/31], [\{\beta = 9/25, g = 14/25\}, 25/2], [\{\beta = 11/20, g = 8/25\}, 100/13], [\{\beta = 11/50, g = 1/100\}, 100/77], [\{\beta = 11/50, g = 21/100\}, 100/57], [\{\beta = 12/25, g = 1/20\}, 100/47], [\{\beta = 14/25, g = 3/100\}, 100/41], [\{\beta = 17/100, g = 7/50\}, 100/69], [\{\beta = 19/100, g = 9/20\}, 25/9], [\{\beta = 19/100, g = 23/100\}, 50/29], [\{\beta = 23/100, g = 1/5\}, 100/57], [\{\beta = 23/100, g = 1/25\}, 100/73], [\{\beta = 39/100, g = 7/50\}, 100/47], [\{\beta = 43/100, g = 39/100\}, 50/9],$$

The parameter values and equilibrium for which the $K = 3$ are:

$$[\{\beta = 17/20, g = 1/50\}, 100/13], [\{\beta = 31/50, g = 6/25\}, 50/7], [\{\beta = 67/100, g = 1/5\}, 100/13], [\{\beta = 73/100, g = 2/25\}, 100/19],$$

The parameter values and equilibrium for which the $K = 4$ are:

$$[\{\beta = 19/50, g = 57/100\}, 20], [\{\beta = 43/50, g = 3/100\}, 100/11], [\{\beta = 73/100, g = 9/50\}, 100/9],$$

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31 $\quad \dfrac{1}{4}\dfrac{M^2-q^2+4x_n+4(q-1)x_{n-1}}{x_n}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 + 4x_n + 4(q-1)x_{n-1}}{x_n}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$$[\{M = 15/2, q = 201/100\}, 951/200], [\{M = 22/5, q = 183/50\}, 403/100], [\{M = 29/4, q = 29/5\}, 261/40], [\{M = 34/5, q = 253/50\}, 593/100], [\{M = 47/5, q = 191/25\}, 213/25], [\{M = 77/10, q = 191/25\}, 767/100], [\{M = 91/25, q = 71/25\}, 81/25], [\{M = 97/25, q = 69/20\}, 733/200], [\{M = 134/25, q = 279/100\}, 163/40], [\{M = 136/25, q = 221/100\}, 153/40], [\{M = 163/25, q = 237/100\}, 889/200], [\{M = 167/20, q = 211/100\}, 523/100], [\{M = 172/25, q = 207/100\}, 179/40], [\{M = 196/25, q = 127/50\}, 519/100], [\{M = 217/50, q = 57/20\}, 719/200], [\{M = 231/50, q = 431/100\}, 893/200], [\{M = 319/50, q = 149/50\}, 117/25], [\{M = 339/50, q = 313/100\}, 991/200], [\{M = 369/100, q = 33/10\}, 699/200], [\{M = 371/50, q = 259/50\}, 63/10], [\{M = 383/100, q = 117/50\}, 617/200], [\{M = 407/50, q = 729/100\}, 1543/200], [\{M = 431/100, q = 311/100\}, 371/100], [\{M = 477/100, q = 279/100\}, 189/50], [\{M = 479/50, q = 119/20\}, 1553/200], [\{M = 497/50, q = 317/50\}, 407/50], [\{M = 497/50, q = 359/50\}, 214/25], [\{M = 527/100, q = 127/25\}, 207/40], [\{M = 579/100, q = 289/100\}, 217/50], [\{M = 647/100, q = 59/10\}, 1237/200], [\{M = 649/100, q = 419/100\}, 267/50], [\{M = 673/100, q = 61/20\}, 489/100], [\{M = 691/100, q = 589/100\}, 32/5], [\{M = 693/100, q = 102/25\}, 1101/200], [\{M = 717/100, q = 317/50\}, 1351/200], [\{M = 733/100, q = 89/20\}, 589/100], [\{M = 769/100, q = 147/20\}, 188/25], [\{M = 801/100, q = 399/50\}, 1599/200], [\{M = 873/100, q = 202/25\}, 1681/200], [\{M = 891/100, q = 123/25\}, 1383/200], [\{M = 907/100, q = 263/100\}, 117/20], [\{M = 937/100, q = 663/100\}, 8], [\{M = 977/100, q = 109/25\}, 1413/200],$$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$$[\{M = 23/4, q = 5/4\}, 7/2], [\{M = 96/25, q = 27/25\}, 123/50], [\{M = 203/50, q = 199/100\}, 121/40], [\{M = 211/100, q = 179/100\}, 39/20], [\{M = 261/50, q = 173/100\}, 139/40], [\{M = 393/50, q = 171/100\}, 957/200], [\{M = 931/100, q = 129/100\}, 53/10],$$

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32 $\quad \frac{1}{4}\frac{M^2-q^2+4(q-1)x_n+4x_{n-1}}{x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 + 4(q-1)x_n + 4x_{n-1}}{x_{n-1}}$$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$$[\{M = 26/5, q = 497/100\}, 1017/200], [\{M = 43/5, q = 173/50\}, 603/100], [\{M = 79/10, q = 158/25\}, 711/100], [\{M = 154/25, q = 113/100\}, 729/200], [\{M = 167/20, q = 719/100\}, 777/100], [\{M = 171/25, q = 13/10\}, 407/100], [\{M = 176/25, q = 259/50\}, 611/100], [\{M = 211/25, q = 79/10\}, 817/100], [\{M = 219/25, q = 51/50\}, 489/100], [\{M = 228/25, q = 177/25\}, 81/10], [\{M = 231/50, q = 229/50\}, 23/5], [\{M = 237/25, q = 749/100\}, 1697/200], [\{M = 238/25, q = 629/100\}, 1581/200], [\{M = 239/25, q = 79/20\}, 1351/200], [\{M = 433/50, q = 88/25\}, 609/100], [\{M = 461/100, q = 31/25\}, 117/40], [\{M = 469/100, q = 83/50\}, 127/40], [\{M = 551/100, q = 43/20\}, 383/100], [\{M = 611/100, q = 151/25\}, 243/40], [\{M = 633/100, q = 293/50\}, 1219/200], [\{M = 671/100, q = 623/100\}, 647/100], [\{M = 709/100, q = 617/100\}, 663/100], [\{M = 769/100, q = 171/100\}, 47/10], [\{M = 831/100, q = 129/20\}, 369/50], [\{M = 961/100, q = 923/100\}, 471/50],$$

The parameter values and equilibrium for which the $K = 5$ are:

$$[\{M = 21/5, q = 257/100\}, 677/200], [\{M = 43/5, q = 429/100\}, 1289/200], [\{M = 46/5, q = 461/100\}, 1381/200], [\{M = 69/20, q = 153/50\}, 651/200], [\{M = 79/10, q = 367/100\}, 1157/200], [\{M = 103/50, q = 187/100\}, 393/200], [\{M = 118/25, q = 189/100\}, 661/200], [\{M = 124/25, q = 267/100\}, 763/200], [\{M = 127/50, q = 89/50\}, 54/25], [\{M = 137/20, q = 16/5\}, 201/40], [\{M = 193/25, q = 18/5\}, 283/50], [\{M = 211/50, q = 9/5\}, 301/100], [\{M = 273/100, q = 63/25\}, 21/8], [\{M = 329/100, q =$$

$149/50\}, 627/200], [\{M = 343/100, q = 52/25\}, 551/200], [\{M = 347/50, q = 409/100\}, 1103/200], [\{M = 363/100, q = 37/20\}, 137/50], [\{M = 423/50, q = 111/20\}, 1401/200], [\{M = 547/100, q = 19/5\}, 927/200], [\{M = 629/100, q = 289/100\}, 459/100], [\{M = 751/100, q = 399/100\}, 23/4], [\{M = 831/100, q = 134/25\}, 1367/200], [\{M = 887/100, q = 521/100\}, 176/25], [\{M = 981/100, q = 6\}, 1581/200], [\{M = 999/100, q = 599/100\}, 799/100],$

Finished investigating difference equation 32 out of 39

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33 $\dfrac{1}{4}\dfrac{M^2r^2-q^2r^2+4rx_n}{qr+r+x_n(1-r)+rx_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2r^2 - q^2r^2 + 4rx_n}{qr + r + x_n(1-r) + rx_{n-1}}$$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 24/5, q = 113/25, r = 39/50\}, 273/2500], [\{M = 27/5, q = 437/100, r = 47/100\}, 4841/20000], [\{M = 32/5, q = 62/25, r = 7/100\}, 343/2500], [\{M = 77/10, q = 158/25, r = 18/25\}, 621/1250], [\{M = 79/20, q = 237/100, r = 24/25\}, 474/625], [\{M = 92/25, q = 121/50, r = 11/20\}, 693/2000], [\{M = 97/25, q = 129/50, r = 33/100\}, 429/2000], [\{M = 123/25, q = 311/100, r = 41/50\}, 7421/10000], [\{M = 124/25, q = 29/100, r = 18/25\}, 4203/2500], [\{M = 127/25, q = 233/50, r = 3/10\}, 63/1000], [\{M = 138/25, q = 239/50, r = 39/50\}, 1443/5000], [\{M = 151/100, q = 37/100, r = 11/25\}, 627/2500], [\{M = 156/25, q = 289/50, r = 3/50\}, 69/5000], [\{M = 172/25, q = 79/20, r = 13/25\}, 3809/5000], [\{M = 183/100, q = 129/100, r = 31/100\}, 837/10000], [\{M = 189/20, q = 9/5, r = 17/50\}, 2601/2000], [\{M = 192/25, q = 281/100, r = 11/25\}, 5357/5000], [\{M = 239/100, q = 43/100, r = 91/100\}, 4459/5000], [\{M = 243/50, q = 419/100, r = 3/4\}, 201/800], [\{M = 299/50, q = 137/100, r = 6/25\}, 1383/2500], [\{M = 321/100, q = 19/100, r = 13/50\}, 1963/5000], [\{M = 367/50, q = 587/100, r = 37/100\}, 5439/20000], [\{M = 397/100, q = 13/20, r = 27/100\}, 2241/5000], [\{M = 399/50, q = 126/25, r = 29/50\}, 4263/5000], [\{M = 417/50, q = 119/50, r = 1/2\}, 149/100], [\{M = 441/100, q = 88/25, r = 1/20\}, 89/4000], [\{M = 513/100, q = 313/100, r = 13/20\}, 13/20], [\{M = 547/100, q = 113/25, r = 9/50\}, 171/2000], [\{M = 551/100, q = 24/25, r = 43/100\}, 3913/4000], [\{M = 621/100, q = 103/50, r = 6/25\}, 249/500], [\{M = 637/100, q = 79/100, r = 49/100\}, 13671/10000], [\{M = 659/100, q = 13/5, r = 11/50\}, 4389/10000], [\{M = 671/100, q = 61/20, r = 19/50\}, 3477/5000], [\{M = 673/100, q = 82/25, r = 49/100\}, 3381/4000], [\{M = 677/100, q = 79/50, r = 47/50\}, 24393/10000], [\{M = 691/100, q = 59/100, r = 49/100\}, 3871/2500], [\{M = 697/100, q = 279/50, r = 7/50\}, 973/10000], [\{M = $

$713/100, q = 69/25, r = 91/100\}, 39767/20000], [\{M = 767/100, q = 89/20, r = 9/10\}, 1449/1000], [\{M = 777/100, q = 653/100, r = 1/20\}, 31/1000], [\{M = 801/100, q = 399/100, r = 16/25\}, 804/625], [\{M = 807/100, q = 79/25, r = 21/100\}, 10311/20000], [\{M = 851/100, q = 71/20, r = 41/100\}, 1271/1250], [\{M = 851/100, q = 713/100, r = 29/100\}, 2001/10000], [\{M = 891/100, q = 861/100, r = 1\}, 3/20], [\{M = 951/100, q = 183/25, r = 3/4\}, 657/800], [\{M = 999/100, q = 813/100, r = 9/20\}, 837/2000],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{M = 233/25, q = 13/50, r = 27/50\}, 12231/5000], [\{M = 903/100, q = 41/25, r = 1/50\}, 739/10000],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 311/50, q = 41/100, r = 43/50\}, 24983/10000],$

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34 $\dfrac{1}{4}\dfrac{M^2r^2-q^2r^2+4rx_{n-1}}{qr+r+x_n(1-r)+rx_{n-1}},$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2r^2 - q^2r^2 + 4rx_{n-1}}{qr + r + x_n(1 - r) + rx_{n-1}}$$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 59/10, q = 37/100, r = 31/50\}, 17143/10000], [\{M = 91/25, q = 119/100, r = 3/5\}, 147/200], [\{M = 103/25, q = 73/50, r = 43/50\}, 5719/5000], [\{M = 118/25, q = 29/100, r = 67/100\}, 29681/20000], [\{M = 126/25, q = 279/100, r = 39/100\}, 351/800], [\{M = 127/25, q = 227/100, r = 81/100\}, 22761/20000], [\{M = 141/25, q = 51/50, r = 8/25\}, 462/625], [\{M = 154/25, q = 97/25, r = 13/20\}, 741/1000], [\{M = 161/20, q = 83/25, r = 61/100\}, 28853/20000], [\{M = 166/25, q = 13/5, r = 49/50\}, 4949/2500], [\{M = 174/25, q = 617/100, r = 1/4\}, 79/800], [\{M = 176/25, q = 82/25, r = 97/100\}, 4559/2500], [\{M = 177/50, q = 39/100, r = 49/50\}, 3087/2000], [\{M = 224/25, q = 99/25, r = 91/100\}, 91/40], [\{M = 237/25, q = 421/100, r = 47/100\}, 24769/20000], [\{M = 247/100, q = 61/100, r = 14/25\}, 651/1250], [\{M = 257/50, q = 114/25, r = 77/100\}, 2233/10000], [\{M = 257/100, q = 13/10, r = 41/50\}, 5207/10000], [\{M = 259/50, q = 66/25, r = 9/50\}, 1143/5000], [\{M = 297/50, q = 487/100, r = 6/25\}, 321/2500], [\{M = 361/100, q = 191/100, r = 12/25\}, 51/125], [\{M = 369/100, q = 11/25, r = 23/25\}, 299/200], [\{M =$

$381/50, q = 119/50, r = 18/25\}, 1179/625], [\{M = 403/50, q = 167/100, r = 29/100\}, 18531/20000], [\{M = 413/100, q = 57/20, r = 79/100\}, 316/625], [\{M = 423/100, q = 11/100, r = 31/100\}, 3193/5000], [\{M = 423/100, q = 97/50, r = 24/25\}, 687/625], [\{M = 457/50, q = 173/20, r = 31/50\}, 1519/10000], [\{M = 457/100, q = 143/100, r = 79/100\}, 12403/10000], [\{M = 471/100, q = 27/100, r = 63/100\}, 6993/5000], [\{M = 587/100, q = 409/100, r = 81/100\}, 7209/10000], [\{M = 617/100, q = 363/100, r = 22/25\}, 1397/1250], [\{M = 631/100, q = 471/100, r = 1\}, 4/5], [\{M = 671/100, q = 59/50, r = 49/50\}, 27097/10000], [\{M = 871/100, q = 347/50, r = 31/100\}, 5487/20000], [\{M = 879/100, q = 99/25, r = 7/100\}, 3381/20000], [\{M = 893/100, q = 479/100, r = 7/20\}, 1449/2000], [\{M = 933/100, q = 839/100, r = 91/100\}, 4277/10000], [\{M = 949/100, q = 907/100, r = 11/100\}, 231/10000], [\{M = 979/100, q = 473/50, r = 17/25\}, 561/5000],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{M = 97/10, q = 223/100, r = 9/100\}, 6723/20000], [\{M = 187/20, q = 1/25, r = 13/50\}, 12103/10000], [\{M = 871/100, q = 23/25, r = 17/50\}, 13243/10000],$

The parameter values and equilibrium for which the $K = 5$ are:

$[\{M = 177/25, q = 8/25, r = 2/25\}, 169/625],$

Finished investigating difference equation 34 out of 39

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35 $\quad \frac{\beta x_n + x_{n-1}}{A + B x_n + x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{\beta x_n + x_{n-1}}{A + B x_n + x_{n-1}}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{A = 3/5, B = 51/50, \beta = 577/100\}, 0], [\{A = 13/10, B = 277/50, \beta = 136/25\}, 0], [\{A = 13/50, B = 101/100, \beta = 29/10\}, 0], [\{A = 34/25, B = 91/10, \beta = 299/100\}, 0], [\{A = 41/50, B = 181/25, \beta = 841/100\}, 0], [\{A = 51/25, B = 699/100, \beta = 269/50\}, 0], [\{A = 71/100, B = 25/4, \beta = 767/100\}, 0], [\{A = 77/20, B = 101/50, \beta = 77/20\}, 0], [\{A = 91/50, B = 16/25, \beta = 49/50\}, 0], [\{A = 93/20, B = 119/100, \beta = 209/25\}, 0], [\{A = 99/100, B = 609/100, \beta = 3/5\}, 0], [\{A = 137/100, B = 117/100, \beta = 57/25\}, 0], [\{A = 149/20, B = 891/100, \beta = 867/100\}, 0], [\{A = 163/20, B = 13/5, \beta = 359/50\}, 0], [\{A = 171/100, B = 69/50, \beta = 4/5\}, 0], [\{A = 201/25, B = 46/5, \beta = 232/25\}, 0], [\{A = 251/50, B = 299/50, \beta = 181/20\}, 0], [\{A = 283/100, B = 329/50, \beta = 363/100\}, 0], [\{A =$

$291/100, B = 281/100, \beta = 293/100\}, 0], [\{A = 343/100, B = 323/50, \beta = 281/100\}, 0], [\{A = 381/50, B = 273/100, \beta = 397/50\}, 0], [\{A = 419/50, B = 33/20, \beta = 791/100\}, 0], [\{A = 501/100, B = 27/25, \beta = 939/100\}, 0], [\{A = 501/100, B = 449/100, \beta = 297/50\}, 0], [\{A = 503/100, B = 133/20, \beta = 423/50\}, 0],$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{A = 7/2, B = 151/20, \beta = 99/100\}, 0], [\{A = 34/5, B = 2, \beta = 171/50\}, 0], [\{A = 57/10, B = 148/25, \beta = 229/50\}, 0], [\{A = 63/10, B = 731/100, \beta = 441/100\}, 0], [\{A = 77/10, B = 427/100, \beta = 29/5\}, 0], [\{A = 77/20, B = 101/50, \beta = 77/20\}, 50/151], [\{A = 91/50, B = 16/25, \beta = 49/50\}, 4/41], [\{A = 131/20, B = 331/50, \beta = 28/25\}, 0], [\{A = 149/20, B = 891/100, \beta = 867/100\}, 222/991], [\{A = 149/50, B = 249/100, \beta = 8/5\}, 0], [\{A = 151/20, B = 337/100, \beta = 49/50\}, 0], [\{A = 163/20, B = 13/5, \beta = 359/50\}, 1/120], [\{A = 169/25, B = 127/100, \beta = 569/100\}, 0], [\{A = 171/100, B = 69/50, \beta = 4/5\}, 9/238], [\{A = 191/25, B = 41/20, \beta = 47/10\}, 0], [\{A = 191/50, B = 281/50, \beta = 19/25\}, 0], [\{A = 201/25, B = 46/5, \beta = 232/25\}, 56/255], [\{A = 214/25, B = 19/25, \beta = 619/100\}, 0], [\{A = 247/25, B = 1/25, \beta = 153/25\}, 0], [\{A = 247/25, B = 173/20, \beta = 11/25\}, 0], [\{A = 271/50, B = 477/100, \beta = 9/4\}, 0], [\{A = 321/100, B = 741/100, \beta = 2/25\}, 0], [\{A = 343/100, B = 323/50, \beta = 281/100\}, 19/373], [\{A = 381/50, B = 273/100, \beta = 397/50\}, 132/373], [\{A = 419/50, B = 33/20, \beta = 791/100\}, 1/5], [\{A = 453/100, B = 321/50, \beta = 1\}, 0], [\{A = 471/50, B = 222/25, \beta = 379/50\}, 0], [\{A = 501/100, B = 449/100, \beta = 297/50\}, 193/549], [\{A = 549/100, B = 9/50, \beta = 411/100\}, 0], [\{A = 657/100, B = 2/5, \beta = 267/100\}, 0], [\{A = 663/100, B = 56/25, \beta = 54/25\}, 0], [\{A = 723/100, B = 247/25, \beta = 148/25\}, 0], [\{A = 877/100, B = 239/100, \beta = 103/50\}, 0], [\{A = 959/100, B = 24/25, \beta = 111/25\}, 0], [\{A = 963/100, B = 13/100, \beta = 289/50\}, 0],$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{A = 3/5, B = 51/50, \beta = 577/100\}, 617/202], [\{A = 13/10, B = 277/50, \beta = 136/25\}, 257/327], [\{A = 13/50, B = 101/100, \beta = 29/10\}, 364/201], [\{A = 34/25, B = 91/10, \beta = 299/100\}, 263/1010], [\{A = 41/50, B = 181/25, \beta = 841/100\}, 859/824], [\{A = 51/25, B = 699/100, \beta = 269/50\}, 434/799], [\{A = 71/100, B = 25/4, \beta = 767/100\}, 796/725], [\{A = 99/100, B = 609/100, \beta = 3/5\}, 61/709], [\{A = 137/100, B = 117/100, \beta = 57/25\}, 191/217],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{A = 283/100, B = 329/50, \beta = 363/100\}, 90/379],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{A = 93/20, B = 119/100, \beta = 209/25\}, 157/73], [\{A = 251/50, B = 299/50, \beta = 181/20\}, 503/698], [\{A = 291/100, B = 281/100, \beta = 293/100\}, 34/127], [\{A = 501/100, B = 27/25, \beta = 939/100\}, 269/104], [\{A = 503/100, B = 133/20, \beta = 423/50\}, 443/765],$

36 $\quad \dfrac{1}{4}\dfrac{M^2-q^2-r^2+2qr+4x_n+4(q-1)x_{n-1}}{r+x_n}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 - r^2 + 2qr + 4x_n + 4(q-1)x_{n-1}}{r + x_n}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{M = 7, q = 511/100, r = 179/100\}, 129/25], [\{M = 9, q = 583/100, r = 13/4\}, 579/100], [\{M = 27/10, q = 543/100, r = 331/100\}, 241/100], [\{M = 48/5, q = 763/100, r = 183/100\}, 77/10], [\{M = 97/10, q = 233/25, r = 17/5\}, 781/100], [\{M = 114/25, q = 173/20, r = 149/25\}, 29/8], [\{M = 123/25, q = 57/20, r = 31/50\}, 143/40], [\{M = 166/25, q = 161/50, r = 4/5\}, 453/100], [\{M = 194/25, q = 959/100, r = 401/100\}, 667/100], [\{M = 323/50, q = 851/100, r = 152/25\}, 889/200], [\{M = 433/50, q = 917/100, r = 129/20\}, 569/100], [\{M = 499/100, q = 389/100, r = 127/100\}, 761/200], [\{M = 541/100, q = 477/100, r = 259/100\}, 759/200], [\{M = 781/100, q = 491/50, r = 629/100\}, 567/100], [\{M = 957/100, q = 779/100, r = 223/50\}, 129/20],$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{M = 4, q = 889/100, r = 733/100\}, 139/50], [\{M = 58/25, q = 261/50, r = 337/100\}, 417/200], [\{M = 78/25, q = 109/25, r = 257/100\}, 491/200], [\{M = 93/100, q = 183/20, r = 491/50\}, 13/100], [\{M = 153/50, q = 551/100, r = 197/50\}, 463/200], [\{M = 347/100, q = 381/50, r = 149/25\}, 513/200], [\{M = 491/50, q = 171/25, r = 529/100\}, 1137/200], [\{M = 533/100, q = 213/50, r = 79/25\}, 643/200], [\{M = 537/100, q = 283/50, r = 467/100\}, 159/50], [\{M = 669/100, q = 491/100, r = 83/25\}, 207/50],$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 4, q = 359/100, r = 172/25\}, 71/200], [\{M = 27/4, q = 453/100, r = 246/25\}, 18/25], [\{M = 34/5, q = 83/25, r = 631/100\}, 381/200], [\{M = 43/5, q = 133/100, r = 29/4\}, 67/50], [\{M = 71/10, q = 311/100, r = 713/100\}, 77/50], [\{M = 123/20, q = 599/100, r = 433/50\}, 87/50], [\{M = 129/25, q = 129/20, r = 659/100\}, 251/100], [\{M = 197/100, q = 132/25, r = 178/25\}, 13/200], [\{M = 207/50, q = 747/100, r = 361/50\}, 439/200], [\{M = 223/25, q = 3/2, r = 191/25\}, 139/100], [\{M = 232/25, q = 259/50, r = 817/100\}, 629/200], [\{M = 233/50, q = 217/50, r = 297/50\}, 153/100], [\{M = 379/50, q = 163/100, r = 613/100\}, 77/50], [\{M = 387/50, q = $

$393/100, r = 172/25\}, 479/200], [\{M = 399/50, q = 148/25, r = 47/5\}, 9/4], [\{M = 463/50, q = 499/100, r = 937/100\}, 61/25], [\{M = 709/100, q = 333/50, r = 224/25\}, 479/200], [\{M = 767/100, q = 247/100, r = 319/50\}, 47/25], [\{M = 831/100, q = 29/25, r = 809/100\}, 69/100], [\{M = 871/100, q = 549/100, r = 477/50\}, 233/100], [\{M = 881/100, q = 86/25, r = 23/5\}, 153/40],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 189/25, q = 106/25, r = 393/100\}, 787/200], [\{M = 197/25, q = 419/50, r = 909/100\}, 717/200], [\{M = 301/50, q = 297/50, r = 563/100\}, 633/200], [\{M = 491/100, q = 124/25, r = 113/25\}, 107/40],$

Finished investigating difference equation 36 out of 39

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37 $\quad \dfrac{1}{4}\dfrac{M^2 - q^2 - r^2 + 2qr + 4(q-1)x_n + 4x_{n-1}}{r + x_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4}\frac{M^2 - q^2 - r^2 + 2qr + 4(q-1)x_n + 4x_{n-1}}{r + x_{n-1}},$$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$$[\{M = 719/100, q = 703/100, r = 141/100\}, 1281/200],$$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 15/4, q = 61/10, r = 433/50\}, 119/200], [\{M = 89/50, q = 113/50, r = 161/50\}, 41/100], [\{M = 94/25, q = 214/25, r = 46/5\}, 39/25], [\{M = 113/20, q = 419/100, r = 817/100\}, 167/200], [\{M = 127/50, q = 178/25, r = 719/100\}, 247/200], [\{M = 149/50, q = 157/25, r = 9\}, 13/100], [\{M = 161/25, q = 599/100, r = 383/50\}, 477/200], [\{M = 162/25, q = 201/100, r = 367/100\}, 241/100], [\{M = 246/25, q = 11/2, r = 149/25\}, 469/100], [\{M = 249/50, q = 104/25, r = 187/25\}, 83/100], [\{M = 281/100, q = 199/100, r = 359/100\}, 121/200], [\{M = 327/50, q = 49/20, r = 222/25\}, 11/200], [\{M = 367/100, q = 44/25, r = 76/25\}, 239/200], [\{M = 481/50, q = 453/100, r = 173/20\}, 11/4], [\{M = 511/100, q = 607/100, r = 967/100\}, 151/200], [\{M = 573/100, q = 243/100, r = 779/100\}, 37/200], [\{M = 617/100, q = 433/100, r = 641/100\}, 409/200], [\{M = 623/100, q = 141/100, r = 237/100\}, 527/200], [\{M = 719/100, q = 217/25, r = 983/100\}, 151/50], [\{M = 827/100, q = 751/100, r = 173/20\}, 713/200], [\{M = 931/100, q = 146/25, r = 997/100\}, 259/100], [\{M = 951/100, q = 541/100, r = 149/20\}, 747/200], [\{M = 967/100, q = 713/100, r = 197/20\}, 139/40], [\{M = 977/100, q = 583/100, r = 79/10\}, 77/20], [\{M = 979/100, q = 573/100, r = 34/5\}, 109/25],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{M = 41/10, q = 307/50, r = 487/100\}, 537/200], [\{M = 53/100, q = 84/25, r = 381/100\}, 1/25], [\{M = 54/25, q = 877/100, r = 417/50\}, 259/200], [\{M = 93/25, q = 439/50, r = 611/100\}, 639/200], [\{M = 119/20, q = 683/100, r = 133/25\}, 373/100], [\{M = 141/20, q = 659/100, r = 551/100\}, 813/200], [\{M = 231/50, q = 114/25, r = 91/25\}, 277/100], [\{M = 241/50, q = 137/25, r = 239/100\}, 791/200], [\{M = 243/50, q = 11/5, r = 47/25\}, 259/100], [\{M = 379/100, q = 41/5, r = 351/50\}, 497/200], [\{M = 401/50, q = 188/25, r = 363/50\}, 207/50], [\{M = 607/100, q = 329/50, r = 419/100\}, 423/100], [\{M = 791/100, q = 39/5, r = 132/25\}, 1043/200],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 191/25, q = 79/10, r = 82/25\}, 613/100], [\{M = 223/50, q = 6, r = 5/2\}, 199/50], [\{M = 297/100, q = 451/50, r = 667/100\}, 133/50], [\{M = 297/100, q = 809/100, r = 134/25\}, 57/20],$

The parameter values and equilibrium for which the $K = 5$ are:

$[\{M = 46/5, q = 453/100, r = 5/2\}, 1123/200], [\{M = 139/25, q = 583/100, r = 213/100\}, 463/100], [\{M = 154/25, q = 31/10, r = 29/20\}, 781/200], [\{M = 178/25, q = 163/25, r = 27/20\}, 1229/200], [\{M = 179/20, q = 25/4, r = 51/20\}, 253/40], [\{M = 453/50, q = 637/100, r = 29/25\}, 1427/200], [\{M = 821/100, q = 559/100, r = 9/4\}, 231/40],$

Finished investigating difference equation 37 out of 39

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38 $\quad \dfrac{1}{4} \dfrac{M^2 r^2 - q^2 r^2 + 4r x_n + 4r x_{n-1}(q-1)}{r x_n + x_{n-1}(1-r)}$

For the rational difference equation

$$\frac{1}{4} \frac{M^2 r^2 - q^2 r^2 + 4r x_n + 4r x_{n-1}(q-1)}{r x_n + x_{n-1}(1-r)}$$

The parameter values and equilibrium for which the equilibrium is not LAS are:

$[\{M = 177/25, q = 671/100, r = 41/50\}, 56539/10000], [\{M = 451/50, q = 781/100, r = 19/25\}, 31977/5000],$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{M = 10, q = 196/25, r = 67/100\}, 14941/2500], [\{M = 297/100, q = 34/25, r = 1/5\}, 433/1000],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{M = 37/4, q = 119/50, r = 11/25\}, 12793/5000], [\{M = 167/20, q = 379/50, r = 23/50\}, 36639/10000], [\{M = 183/25, q = 351/100, r = 11/20\}, 11913/4000], [\{M = 219/50, q = 119/50, r = 47/100\}, 7943/5000], [\{M = 369/50, q = 133/100, r = 69/100\}, 60099/20000], [\{M = 409/100, q = 307/100, r = 8/25\}, 716/625], [\{M = 421/50, q = 53/50, r = 37/50\}, 8769/2500], [\{M = 487/50, q = 973/100, r = 19/100\}, 36993/20000], [\{M = 799/100, q = 53/50, r = 19/25\}, 3439/1000], [\{M = 877/100, q = 233/50, r = 23/50\}, 30889/10000], [\{M = 889/100, q = 97/50, r = 87/100\}, 94221/20000], [\{M = 951/100, q = 59/10, r = 43/100\}, 66263/20000],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 9, q = 613/100, r = 3/100\}, 4539/20000], [\{M = 42/5, q = 799/100, r = 27/50\}, 44253/10000], [\{M = 167/50, q = 113/100, r = 9/20\}, 4023/4000], [\{M = 176/25, q = 249/50, r = 11/100\}, 6611/10000], [\{M = 203/50, q = 3/2, r = 1/5\}, 139/250], [\{M = 339/50, q = 37/20, r = 17/100\}, 14671/20000], [\{M = 687/100, q = 99/25, r = 73/100\}, 79059/20000],$

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39 $\quad \dfrac{1}{4} \dfrac{M^2 r^2 - q^2 r^2 - s^2 r + 2qrs + 4rx_n(q-1) + 4x_{n-1}}{sr + x_n(1-r) + rx_{n-1}}$

For the rational difference equation

$$x_{n+1} = \frac{1}{4} \frac{M^2 r^2 - q^2 r^2 - s^2 r + 2qrs + 4rx_n(q-1) + 4x_{n-1}}{sr + x_n(1-r) + rx_{n-1}}$$

The parameter values and equilibrium for which the MaxK value, 5, , is not high enough are:

$[\{M = 7/10, q = 749/100, r = 41/100, s = 753/100\}, 1353/10000,], [\{M = 641/100, q = 101/100, r = 2/25, s = 3/5\}, 341/1250],$

The parameter values and equilibrium for which the $K = 2$ are:

$[\{M = 13/2, q = 238/25, r = 19/20, s = 247/25\}, 5833/2000,], [\{M = 32/5, q = 341/50, r = 7/100, s = 417/50\}, 427/2500], [\{M = 39/20, q = 197/50, r = 79/100, s = 411/100\}, 7031/10000,], [\{M = 44/5, q = 183/50, r = 17/100, s = 83/50\}, 459/500], [\{M = 48/5, q = 16/5, r = 11/25, s = 459/50\}, 1991/2500], [\{M = 57/10, q = 447/100, r = 27/100, s = 729/100\}, 243/625,], [\{M = 69/20, q = 106/25, r = 21/25, s = 369/100\}, 42/25], [\{M = 82/25, q = 497/100, r = 81/100, s = 669/100\}, 3159/5000,], [\{M = $

$83/50, q = 261/100, r = 17/100, s = 99/50\}, 3893/20000,], [\{M = 87/25, q = 73/25, r =$
$47/100, s = 491/100\}, 7003/20000,], [\{M = 123/25, q = 133/25, r = 1/50, s =$
$191/20\}, 69/10000,], [\{M = 123/25, q = 241/50, r = 2/25, s = 101/100\}, 873/2500,], [\{M =$
$131/20, q = 649/100, r = 41/100, s = 609/100\}, 5699/4000,], [\{M = 137/20, q =$
$561/100, r = 3/20, s = 151/20\}, 1473/4000,], [\{M = 142/25, q = 299/50, r = 33/50, s =$
$163/25\}, 8481/5000,], [\{M = 199/50, q = 183/100, r = 3/4, s = 69/25\}, 183/160], [\{M =$
$207/100, q = 62/25, r = 27/100, s = 5/2\}, 1107/4000,], [\{M = 217/25, q = 411/50, r =$
$41/100, s = 144/25\}, 22837/10000,], [\{M = 267/50, q = 169/100, r = 23/100, s =$
$283/100\}, 483/1000,], [\{M = 279/50, q = 137/20, r = 12/25, s = 79/50\}, 651/250], [\{M =$
$297/50, q = 91/20, r = 43/100, s = 43/10\}, 26617/20000,], [\{M = 303/50, q = 161/50, r =$
$1/2, s = 81/50\}, 383/200], [\{M = 313/50, q = 33/25, r = 9/100, s =$
$399/100\}, 3231/20000,], [\{M = 387/100, q = 591/100, r = 31/100, s =$
$543/100\}, 2697/4000,], [\{M = 459/50, q = 177/20, r = 9/20, s =$
$101/25\}, 12591/4000,], [\{M = 463/50, q = 3/2, r = 13/50, s = 171/25\}, 637/1250], [\{M =$
$567/100, q = 361/100, r = 77/100, s = 101/20\}, 32571/20000], [\{M = 733/100, q =$
$357/50, r = 7/20, s = 236/25\}, 3521/4000,], [\{M = 749/100, q = 32/5, r = 17/100, s =$
$421/100\}, 2057/2500,], [\{M = 821/100, q = 198/25, r = 59/100, s =$
$202/25\}, 9499/4000,], [\{M = 961/100, q = 879/100, r = 22/25, s = 27/4\}, 2563/500,],$

The parameter values and equilibrium for which the $K = 3$ are:

$[\{M = 8, q = 487/50, r = 79/100, s = 108/25\}, 53009/10000,], [\{M = 191/50, q =$
$351/100, r = 24/25, s = 58/25\}, 1503/625,], [\{M = 337/100, q = 961/100, r = 3/5, s =$
$843/100\}, 273/200,], [\{M = 909/100, q = 27/10, r = 13/50, s =$
$131/100\}, 1703/1250,], [\{M = 961/100, q = 137/25, r = 17/50, s = 17/25\}, 24497/10000,],$

The parameter values and equilibrium for which the $K = 4$ are:

$[\{M = 32/5, q = 234/25, r = 1, s = 339/100\}, 1237/200],$

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