Syllabus: Math 508, Spring 2023

The course will cover the following subjects:

1. Dunford spectral calculus for analytic functions.
2. Gelfand's theory of Banach algebras and *-algebras.
3. The spectrum of a bounded operator on a Banach space.
4. The spectral resolution for bounded normal operators on a Hilbert space.
5. Unbounded self-adjoint operators.
6. Basic properties of distributions.
7. Sobolev spaces in $\mathbb{R}^n$, Sobolev embeddings, compactness, extension, trace.
8. Solving basic PDE using functional analysis tools.

It will also cover some subset of the following additional topics, depending on time and interest:

1. Flows and semigroups.
2. Nonlinear problems, fixed point arguments, linearization.
3. Perturbation of operators.
4. The Fourier transform.
5. Spectral theory for Schrödinger equations.
6. Invariant subspaces.

There will be no exams. Some problems will be given out from time to time. Grades will be based on presentations on some of the additional topics above, or other relevant material.

Texts

There is no required textbook, nor will I be following any textbook closely (at least not for the entire course).

For a review of the content from the previous semester, consult Haim Brezis, *Functional Analysis, Sobolev Spaces, and Partial Differential Equations*. For a reasonable text with good examples to read alongside the discussion of spectral theory, I recommend Peter Lax, *Functional Analysis*. For a comprehensible take on Sobolev spaces and their use in solving PDE, I recommend either Brezis or Craig Evans, *Partial Differential Equations* (this book contains many other things; for the purposes of this course only Part II will be relevant). We will go into more detail than either of these texts do, but they are at least readable and worthwhile to follow along with. I also mention other texts below, but I cannot in good faith say the same about them. I may supply some additional notes if I feel they would be needed.

For a more comprehensive treatment of spectral theory, see N. Dunford and J. Schwartz, *Linear Operators* (particularly Vol. II on Hilbert spaces). The classic reference on the theory of distributions is Lars Hörmander, *The Analysis of Linear Partial Differential Operators*; while this is a four-volume series, generally only the first is relevant to non-specialists. We will not go into nearly the level of detail presented there, however. For Sobolev spaces in general, particularly on domains, Vladimir Maz'ya, *Sobolev Spaces: with Applications to Elliptic Partial Differential Equations* is in my opinion often superior to other modern texts, but uses nonstandard notation and can be difficult to read. The more standard reference is Robert Adams, *Sobolev Spaces*.