Some useful formulas in and around linear algebra

Rotations: $A_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Values of Trigonometric functions

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Linear systems

Consistency of $A\mathbf{x} = \mathbf{b}$: $\mathbf{b}$ is in the column space

Back and forth substitution: $L\mathbf{y} = \mathbf{b}; U\mathbf{x} = \mathbf{y}; LU\mathbf{x} = \mathbf{b}$

$U = E_k \cdots E_1 A$: $L = E_1^{-1} \cdots E_k^{-1}$

Matrix algebra

$(AB)^{-1} = B^{-1}A^{-1}$ $\qquad$ $(P^{-1}DP)^n = P^{-1}D^nP$
$\det(AB) = \det(A)\det(B)$

Inversion Algorithm: $[A \quad I] \to [I \quad A^{-1}]$

To write an invertible matrix A as a product of elementary matrices:
(1) $E_k \cdots E_1 A = I$ (row reduction to reduced row echelon form);
(2) $A = E_1^{-1} \cdots E_k^{-1}$.
Invertibility: $\det A \neq 0$

Basis algorithms

1. Row space: nonzero rows of row reduced form. Dimension: rank.
2. Column space: pivot columns of original matrix. Dimension: rank.
3. Null space: one basis vector per free variable: set one free variable equal to 1, set the others equal to 0, and solve for the basic variables. Dimension: nullity.

Eigenvalues

Characteristic polynomial: $\det(A - \lambda I)$

Diagonalization and Power algorithms

$P^{-1}AP = D$: P is the matrix of eigenvectors, D is the matrix of eigenvalues.
$A^n = PD^nP^{-1}$

Dot products and Projections

$\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T\mathbf{v} = ||\mathbf{u}||||\mathbf{v}|| \cos\theta$
$||\mathbf{u}|| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$
$A\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot A^T\mathbf{v}$ (adjoint property)
Projection of $\mathbf{v}$ in the direction of $\mathbf{u}$: $\dfrac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u}$