Introduction to Differential Equations in Mathematica

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1 Using DSolve

We will start by solving the following ODE:

$$y''(t) = 3y(t)$$

Type the following code to have Mathematica solve this equation:

```
sol = DSolve[y''[t] = = 3 y[t],y,t]
```

This will return a list of rules. To assign a function f to one of these solutions, type the following syntax:

```
f[t_] = y[t]/.sol[[1]]
```

This will set f to be the function defined by y in the first rule listed in sol.

However, we aren't done yet. We still have to assign numbers to the constants C[1] and C[2]. To do this, we follow similar syntax as above:

```
F[t_] = f[t]/.{C[1] -> 1, C[2] -> 2}
```

Plot this function on the interval $[-10, 10]$.

Instead of defining the constants after creating the solution, we could have included the initial condition in the DSolve command. For example, if we want to solve the same ODE with the initial condition $y(0) = 1$, we would type

```
sol1 = DSolve[{y''[t] = = 3y[t], y[0]==1}, y[t],t]
```

To plot this as a function, we would again assign a function to the solution:

```
f[t_] = y[t]/.sol1[[1]]
```

2 Tables of Solutions to ODEs

Suppose we wanted to solve the following first order ODE:

$$y'(t) = \left(1 - \frac{y(t)}{100}\right) y(t)$$

Use DSolve to solve the above equation. Assign g to the first solution. Create a table of solutions corresponding to different values of C[1] called tablegparticular by using the following syntax:

```
tablegparticular[t_] = Table[g[t]/.C[1] -> j, {j, 1, 7}]
```

Plot this table of solutions on one graph where t ranges from 0 to 10. You may need to use the Evaluate function, and don't forget to add a key to your plot.

3 Applications of ODEs

Use Mathematica to solve the following ODEs that correspond to the applications indicated. Pick a few initial conditions and plot the graphs.

1. Unbounded population growth with growth rate r: $\frac{dy}{dt} = ry$
2. Population decay with decay rate of r and initial population c: $\frac{dy}{dt} = r(c - y)$
3. Compound interest at a rate of r%: $\frac{dP}{dt} = \frac{r}{100}P$
4. Radioactive Decay at a rate of r: $\frac{dy}{dt} = -ry$
5. Logistic population growth: $\frac{dy}{dt} = (d - cy)(b - ay)$
6. Escape Velocity of an object of small mass m_s orbiting around an object of large mass m_ℓ and gravitational constant G: $m_s \frac{d^2y}{dt^2} = -\frac{Gm_s m_\ell}{y^2}$
7. Planetary Orbits:

$$\left(\frac{H^2}{r^2}\right)\frac{d^2r}{d\theta^2} - \left(\frac{2H^2}{r^3}\right)\left(\frac{dr}{d\theta}\right)^2 = \frac{H^2}{r} - G$$

   Hint: Make the substitution $r = \frac{1}{u}$.

4 Explore!

Search the internet for other interesting ODEs and their applications. See if Mathematica can solve them, and plot the solutions. What happens as we change the initial conditions? What happens as we change the parameters (such as the value of r in many of the above problems)? In your searches, you will likely come across some systems of equations and PDEs (Partial Differential Equations). We will talk about these more in tomorrow's class, but if you want to explore them now I encourage you to look at the Wolfram Language Documentation to find the syntax for solving such problems.