Julia Sets and the Mandelbrot Set in Mathematica

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1 Julia Sets

In this example, we look at the map

$$z \mapsto z^2 + c$$

where c is a complex number. Iterates of this map have two types of behavior:

- Rush off to infinity
- Stay around the origin (actually, near a fixed point)

The two types of iterates are separated by a curve. For $c = 0$, this is the unit circle. As we change the value of c, we run through a range of fractals.

Definition 1.1. The set of points that don't go off to infinity is called the **filled in Julia set**, and the boundary is called the **Julia set**.

Computing the Julia set:

- If an iterate ends up outside $\{z \in \mathbb{C} : |z| \leq 5\}$, then it is not in the Julia set.
- If the first 100 iterates have magnitude less than or equal to 5, then it is deemed to be in the Julia set.

Exercise: Create an algorithm in Mathematica which does the following:

- Inputs: z (an initial point) and c
- Compute the first 100 iterates of z under the map $z \mapsto z^2 + c$.
- If any of the iterates have magnitute greater than 5, output "Yes". Otherwise, output "No".
- Try doing this *without* using an array.

Test your algorithm on the following points:

1. $c = 0, z = e^{i\pi/4}$
2. $c = 0, z = i/4$
3. $c = 0, z = 3$

4. $c = -0.5, z = 0.5i$

5. $c = -0.5, z = -1$

6. $c = -0.5, z = 2$

7. $c = -0.75, z = -0.5$

1.1 Drawing the Julia Set

Mathematica has a built-in function which generates an image of the Julia set for input c-values: JuliaSetPlot[c]. Use this function to generate the Julia sets for the following values of c:

1. $c = 0.1 + 0.05i$
2. $c = -0.6 - 0.3i$
3. $c = -1$
4. $c = -0.1 + 0.15i$
5. $c = 0.25 + 0.52i$
6. $c = -0.5 + 0.55i$
7. $c = 0.66i$
8. $c = -i$

Which of the above c values have Julia sets which look connected (ie all one piece)?

Challenge: Use the manipulate environment to play with the parameters and see the variety of shapes of the Julia set.

2 Mandelbrot Set

We now look at the same map:

$$z \mapsto z^2 + c$$

But this time, we look at the iterates of 0, but vary c. The values of c for which the iterates of 0 do *not* wander off to infinity is called the **Mandelbrot Set**.

Theorem 2.1 (Fundamental Theorem of the Mandelbrot Set)**.** *The point c is in the Mandelbrot set exactly when the Julia set of the function $z \mapsto z^2 + c$ is connected.*

Exercise: Use the built-in Mathematica function MandelbrotSetPlot[] to generate the image of the Mandelbrot set. You are encouraged to play with the parameters of the function to generate different-looking images.

Definition 2.2. Values of c for which the iterates of 0 under the map $z \mapsto z^2 + c$ are eventually periodic are called **Misiurewicz points** after the Polish mathematician Michal Misirewicz.

All Misiurewicz points are on the boundary of the Mandelbrot set. For example, i is a Misiurewicz point. (Compute the iterates of 0 under the map $z \mapsto z^2 + i$.) Points close to the boundary have very strange Julia sets.

Challenge: Create a manipulate environment which allows you to manipulate the real and imaginary parts of c. It should display that point's location on the Mandelbrot set image and generate the image of its Julia set.