Shapes of Constant Width

Are Cirlces the Only Ovals with Constant Width?

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1 Support Functions

Fact: If h is a 2π periodic function satisfying

$$h + h'' > 0 \tag{1}$$

then the function

$$\sigma(\theta) = h(\theta)c(\theta) + h'(\theta)c'(\theta) \tag{2}$$

is an oval with support function h.

Example 1.1. Let $h(\theta) = 3 + \sin(\theta)$. Check that h satisfies equation (1). Plot the curve σ that corresponds to this support function.

Since $W(\theta) = h(\theta) + h(\theta + \pi)$, which we want to be constant, our strategy is to find a 2π periodic function h satisfying:

1. $h(\theta) + h''(\theta) > 0$
2. $h(\theta) + h(\theta + \pi) \equiv c$

One good place to start looking for such functions is by using **odd** functions.

Definition 1.2. A 2π periodic function f is **odd** if $f(\theta + \pi) = -f(\theta)$.

Our strategy moving forward is to set $h(\theta) = c + f(\theta)$ where f is an odd 2π periodic function.

Example 1.3. Use the angle-sum formulas to show that $\sin(\theta)$ and $\cos(\theta)$ are odd functions.

Example 1.4. Let's work with the odd function $\cos(3\theta)$. What values of c can we use so that the function $h(\theta) = c + \cos(3\theta)$ satisfies $h + h'' > 0$?

Plot h with some of the allowable values of c that you computed above. What happens as we grow or shrink c? What if we pick a c that is out of the allowable range that you calculated? Plot an example of this and see what happens.

Now try this for some other odd functions. What kinds of shapes do you get?

Challenge: Create a manipulate environment which is a graph of an oval of constant width for a support function of the form $h(\theta) = c + f(n\theta)$ where n is an odd integer, f is sin or cos, and c is a constant in some range.