## Math-300 Summer 2017

Course Title: Introduction to Mathematical Reasoning
Instructor: Chloe (Urbanski) Wawrzyniak
Instructor Email: ceu11@math.rutgers.edu
Instructor Office and Office Hours: To be determined, based on student feedback
Course Textbook: Book of Proof by Richard Hammack - This is an open source textbook, which is freely available online, and a PDF is posted on our Canvas page. You can also buy a paper copy on Amazon for around \$15, if you so desire.
Course Meeting Times and Location: Mondays and Wednesdays 6-8:30 pm in Tillet 204 - for a detailed course schedule, see below.

Disclaimer: The material in this syllabus is subject to change, according to the instructor's discretion. The exam dates, however, will not change.

## Why this Course is Different

## Why Math-300 is Different from other Math Courses

Most of the Math courses that you've taken up to this point have focused primarily on computation. Often, you would be given problems to solve which could be graded by a computer. This class is not about computation. Rather, it is about the underlying structure of Mathematics that makes that computation possible. In Calculus, you learned various rules for computing the derivative: product rule, power rule, quotient rule, chain rule, etc. Chances are, you took these formulas as given, and memorized when and how to use them. However, the definition of derivative had to do with limits, and says nothing about products or powers. Why, then, do these formulas hold true? Based on the limit definition of a derivative, how do I know that $(\mathrm{fg})^{\prime}=\mathrm{f}^{\prime} \mathrm{g}+\mathrm{g}$ 'f and not $\mathrm{f}^{\prime} \mathrm{g}$ ? The answer is proof. The entire first class will be devoted to a discussion of what is proof, because proof is what this class is all about. It's the glue that holds Mathematics together. It tells us why, when, and how we can make various computations and what the consequences of those computations are. In this class, you will learn how to recognize proof, how to find errors in incorrect proofs, and how to write proofs of your own.

## Why this Section is Different from other Math-300 Courses

Even though Math-300 is different from other Math courses, it is often taught in a similar format: the instructor lectures at the board, and students are sent home with weekly homework to reproduce ideas presented at the board. That will not be the format of this course. Instead, you will be assigned reading from the textbook with a guided worksheet, which will be done outside of class. You will then attend class where you will work in groups on problems based on that reading. At the end of the class, you will be expected to present solutions to problems at the board. I am not planning in lecturing during this course. If, however, the class is struggling with a particular concept, I will jump in with a quick lecture on that topic. Nonetheless, class time will be primarily spent on guided group work. For more details on the structure of the classes and corresponding assignments, see "Grade Computation and Course Components" below.

## Learning Goals

1. Students will be able to identify correct proofs, find and classify errors in incorrect "proofs", and construct correct proofs of their own.
2. Students will be able to identify given text as either an example, a heuristic proof, or a proof, and explain their reasoning.
3. Students will be able to identify and give original examples of basic proof methods, including
4. Direct Proof
5. Contradiction and Contrapositive
6. Proofs with Sets and Quantifiers
7. Induction
8. Pigeonhole Principle
9. Delta-epsilon Proofs
10. Students will be able to translate between English sentences and Mathematical symbols, and will be able to reason using either.
11. Students will be able to construct counterexamples to false statements.
12. Students will be able to compare and contrast proof methods and explain how the methods relate and how they fit into the larger Mathematical context.
13. Students will be prepared to take 311 (Real Analysis), 351 (Abstract Algebra), and other upper level Math courses.

## Academic Integrity

- Group Work: In this class, you will spend a lot of time working in groups, but you will still receive you own grade for the course, and will therefore need to do your own work. So, how do you know when the group work must stop, and the individual work begins? Follow the "Sauna Rule": If you work out problems with other students, do not copy the answers. Instead, go take a 30-minute sauna, and then write up the answer on your own without the aid of group work. Copying work that is not yours is plagiarism, even if the work was done as part of a discussion of a problem.
- Internet Searches and Other Sources: You may not search anything for this class on the internet. You may not use any resources outside of those posted on the Canvas page. You are expected to make mistakes on your homework problems, and the grading policy is designed to compensate for that. I have plenty of office hours where you can ask questions. Searching the internet will often lead you to solutions which are misleading at best. I have posted a lot of resources which you may use, and of which I have confirmed the accuracy. I also have plenty of availability to meet in office hours if you have questions.
- There is one very important exception to this policy: you may (and in fact are encouraged to) search the internet for anything about working in LaTeX. I will not in any way test you on memorization of LaTeX. Professional mathematicians routinely search the internet for help with typesetting, and it would therefore be unreasonable (and quite frankly, pointless) for me to stop you from searching for typesetting help.
- Tutors: You may not use Chegg, Varsity Tutors, or other online tutoring services. You may not use any private tutoring services, online or otherwise. Many available Math tutors are not qualified to help with this course, and I cannot grade you on your reasoning if I don't know how
much of it came from a tutor. The only possible exception to this policy is the on-campus, Rutgers-sponsored learning centers. However, I would prefer you come to office hours first, and if you choose to use the learning centers, you must tell me which assignments you got help with. I should also warn you that there is a very high probability that the tutors at the learning centers will not be able to help with Math-300 course work. If you cannot get the help that you need from class or office hours and would like to hire a private tutor, please come to me so that I can direct you towards graduate students whom I know and trust to help you with your work. This way, there will be no question of academic integrity when grading, and you will know that you are working with someone qualified to help with this course.

If at any point you have questions about these policies, do not hesitate to ask.
Rutgers Academic Integrity Policy (Links to an external site.)

## Disabilities Statement

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation:
https://ods.rutgers.edu/students/documentation-guidelines. If the documentation supports your request for reasonable accommodations, your campus's disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss the accommodations with them as early in your courses as possible. To begin this process, please complete the Registration form on the ODS web site at: https://ods.rutgers.edu/students/registration-form.

## Grade Calculation and Course Components

| Course Component | Percentage of Final Grade |
| :--- | :--- |
| Syllabus Quiz | 2 |
| Quizzes | 3 |
| Homework | 10 |
| In-Class Presentations | 10 |
| Participation | 5 |
| Midterm Exam 1 | 15 |
| Midterm Exam 2 | 15 |
| Final Exam | 30 |
| Portfolio | 10 |

## $\underline{\text { Participation }}$

Participation is worth 5\% of your grade. Every class day (except Exam days) is worth 10 points. You lose 1 point for every 10 minutes that you are late to class. The first time I see you on your cell phone or otherwise off topic outside of breaks, you get a warning. After that, you lose 1 point for each occurrence.

## Quizzes

There will be a total of four quizzes during this semester. Three of those are content quizzes, and their dates and topics are listed in the table below. These each count for $1 \%$ of your grade. The fourth is a quiz on the syllabus and related material (course website, posted assignments and resources, etc). The syllabus quiz is on the Canvas course page. This quiz alone counts for $2 \%$ of your grade. The deadline to finish the quiz is $11: 59 \mathrm{pm}$ on Monday, June 5. You may take this quiz as many times as you like, and the gradebook will keep your highest score. You are allowed and encouraged to reference the syllabus, the course website, and the materials posted while completing this quiz.

| Quiz Topic | Date |
| :--- | :--- |
| Syllabus | Online - Due 11:59 pm Monday, June 5 |
| Contradiction | In Class - Monday, June19 |
| Induction | In Class - Monday, July 10 |
| Cardinality | In Class - Monday, July 24 |

## Homework

Your homework is worth $10 \%$ of your grade. You will have homework after each class, with the exception of exam days. Each homework assignment will consist of two parts - one reviewing the topic covered on the previous day's class, and the other introducing the topic to be covered in the next class.

In the introduction part, you will be given a worksheet with sections of the textbook to read, and questions based on that reading. You should complete the worksheet and then take the corresponding quiz on the Canvas site. You will have one attempt at this quiz, and you may use your notes, completed worksheet, and book while taking the quiz. Though it is listed as a quiz, this is actually the format for how you will submit the introduction homework. You will not turn any homework in during class, as it will all be submitted online. The quiz questions will very closely follow the worksheet questions, so it is in your best interest to complete the worksheet prior to taking the quiz. Pay careful attention to the due dates and times listed on Canvas. These quizzes will close 30 minutes before class, so that correct answers can be released before class starts.

The introduction homework will be graded with 15 points for completeness and 30 points for correctness. You are expected to make mistakes when working through this part of the homework, and the completion points are designed to compensate for this. For the last homework assignment of a module, the intro will instead consist primarily of review problems.

The review part consists of one or more problems directly from the in-class worksheet and problems similar to those on the in-class worksheet. You must write these arguments carefully using complete sentences, and they will be graded on the below rubric. They must be typed and submitted online before the start of class. At the beginning of the course, you have the option to either type directly into Canvas, using its native math-typing capabilities, or you may type in your preferred ${ }^{2} T T_{\mathrm{E}} X_{\text {compiler and upload }}$ the resultant PDF. I strongly prefer the latter, but I will not assume that students are familiar with ${ }^{E A} T_{\mathrm{E}} X_{\text {when entering the class. We will, therefore, spend some time during the semester discussing }}$ compilers and typing in $E T_{\mathrm{E}} X$, so that you can begin practicing. After the first midterm, all of your review homework problems must be typed using $E T_{\mathrm{E}} X$.

Before each exam, you will have the option of rewriting one or two of these review problems, based on the feedback you received in grading. This will be discussed in further detail during class.

## Homework Write-up Rubric

| Category | 5 Points | 3 Points | 1 Point |
| :---: | :---: | :---: | :---: |
| Mathematical Accuracy | No computation errors. All relevant theorems were used and cited properly. No logical errors or missing components. | Overall correct with either few major errors or many smaller errors. These errors may be either computational or logical. | Some correct computation with no corresponding explanation. Alternatively, the correct start or main idea of a proof, without completing the proof (either correctly or at all). |
| Presentation | Typed in ${ }^{E} T_{\mathrm{E}} X_{\text {with correct }}$ formatting and labels. Important equations are given their own line and referenced as needed. | Typed with correct formatting. Equations are not referenced correctly or at all. Some typo's. | Typed, but with little-to-no mathematical formatting (for example, typing the word "theta", instead of the mathematical symbol $\theta$ ). |
| Grammar | Proper English grammar. Typed in paragraphs with complete sentences. Proper punctuation and sentences start with a capital letter. | Explanations provided, but without proper capitalization or punctuation. Not always in complete sentences. | Some words to connect the computations, but never in complete sentences. |

## In-Class Presentations

During a typical class day, students will spend more than half of the class period working on an in-class worksheet in groups. This worksheet will build upon the topics students just completed homework on. During about the last hour of each class, students will be asked to present solutions to the problems on the in-class worksheets or from the homework. Each student is required to present twice throughout the semester. If a student presents more than 2 times, only the 2 highest scores will be kept. These 2 scores will together count for $10 \%$ of your grade.

Each presentation will be scored out of 25 points, using the following rubric. These presentations will be recorded for grading and self-reflection purposes only. The instructor will be the only person with direct access to the recordings, and the students will have access to their own recordings, in order to improve.

## Presentation Rubric

## Statement of Problem (5 Points)

| $\mathbf{0 - 2}$ | $\mathbf{c \|} \mathbf{3}$ | $\mathbf{5}$ |
| :--- | :--- | :--- |
| Incorrect statement of problem. <br> Maybe missing half of an "if and <br> only if" or misinterpret what is given <br> or what is to be shown. | Correct but incomplete statement of <br> the problem. Doesn't include a <br> statement of either the hypotheses or <br> conclusion or fails to connect to the <br> diagram (if applicable) or properly <br> introduce variables. | Correct statement with a labeled <br> diagram and the hypotheses and <br> conclusion stated in terms of the <br> diagram (if applicable), with all <br> variables introduced correctly. |

## Correctness of Proof (15 Points)

| 0-1 | 2-5 | 6-9 | 10-14 | 15 |
| :---: | :---: | :---: | :---: | :---: |
| Mainly incorrect consequences improperly deducted from the given. Little or no sense of how to prove the result. | Unconnected, mostly true statements properly deducted from the given. Listing facts without a sense of how to link them to get a correct proof. Maybe just jumps to the conclusion without justification. | Statements linked to a reasonable (though perhaps misguided) attempt to prove the theorem. The proof may be left incomplete or may depend upon a major unjustified leap. | A correct approach to proving the theorem is attempted. Some statements may be unjustified or improperly justified, but errors are minor and could be fixed given time to polish the proof. | A correct and complete proof is given. Some irrelevant information may be included since the time limit precludes polishing up the presentation. |

Presentation (5 Points)

| $\mathbf{0} \mathbf{0 - 2}$ | $\mathbf{c \|} \mathbf{c \|} \mathbf{c \|} \mathbf{5}$ |  |
| :--- | :--- | :--- |
| Writing is illegible or not <br> adequately used to record <br> information. Does not speak <br> clearly or consistently uses <br> incorrect terminology. | Writing is legible and speaks <br> clearly, but not enough written <br> down or some use of incorrect <br> terminology. | Communicates clearly and <br> effectively. Legible and <br> complete writing. Speaks <br> clearly and effectively using <br> only correct terminology. |

## Exams

There will be two midterm exams, both of which will be during class. They will consist of two parts. The first part will consist of problems similar to the intro homework problems as well as a few short proof problems. The second part will consist of some more involved proof-writing problems. Each midterm exam counts for $15 \%$ of your grade.

The final exam will take place on the last day of class, and will count for $30 \%$ of your final grade. It will consist primarily of writing problems, and will not be split into multiple parts. It will be cumulative.

You must obtain at least a $60 \%$ average on the three exams in order to receive a C in this course. For example, a student who earns exam grades of $50 \%, 60 \%$, and $70 \%$ may still earn a C in the course, since the average of these scores is $60 \%$. The exams are designed so that a student who understands the core material can very well earn above $60 \%$. Therefore, this policy is designed to indicate to you what level of understanding of the material in this course will be required for 351 and 311, and whether you've achieved this level of understanding. You must receive at least a C in this course in order to take 351 and 311.

There will be no makeup midterms. Instead, if you miss a midterm exam, the other midterm will count for $20 \%$ of your grade (instead of $15 \%$ ) and the final will count for $40 \%$ of your grade (instead of $30 \%$ ).

## Final Portfolio

The last component of your course grade is a portfolio, which counts for $10 \%$ of your final grade. You will choose a number of methods from the course that you think are of particular importance, and collect them in one large document. You will explain what the topics are and why you chose them, and give examples (and non-examples) that demonstrate an understanding of the topic. The exact details and grading criteria for this assignment will be discussed more thoroughly in class.

## Course Calendar

This calendar is subject to change, though the exam dates will not change. For a more detailed description of each day's topic and assignments, see the Modules page or the calendar at the bottom of the Syllabus page, both on the Canvas site.

| Date | Topic | Book of Proof: <br> Intro Homework | Book of Proof: In Class Only |
| :---: | :---: | :---: | :---: |
| Wednesday, May 31 | Intro to the Course, What is Proof?, Writing, LaTeX |  | 5.3 |
| Monday, June 5 | Basics of Sets | 1.1-1.6 | 1.7, 1.8 |
| Wednesday, June 7 | Symbolic Logic | 2.1-2.6 | 2.9 |
| Monday, June 12 | Basic Techniques | 4.1-4.5 | 7.1, 7.2 |
| Wednesday, June 14 | Contradiction and Contraposition | 5.1, 6.1, 6.2 | 6.3 |
| Monday, June 19 | Quantifiers | 2.7, 7.3 | 7.4 |
| Wednesday, June 21 | Negation and Disproof | 2.10, 9.1, 9.2 | 9.3 |
| Monday, June 26 | Review | Review Packet |  |
| Wednesday, June 28 | Exam 1 |  |  |
| Monday, July 3 | Heuristics | No Intro Homework |  |
| Wednesday, July 5 | Induction: Part I | 3.4, (Supplement) | 10.3 |
| Monday, July 10 | Proofs with Sets | 8.1-8.3 | 8.4 |
| Wednesday, July 12 | Relations | 11.1, 11.2 | 11.3, 11.4 |
| Monday, July 17 | Functions | 12.1, 12.2, 12.4 | 12.5, 12.6 |
| Wednesday, July 19 | Cardinality: Part I | 3.5, 12.3, 13.1 |  |
| Monday, July 24 | Review | Review Packet |  |
| Wednesday, July $26$ | Exam 2 |  |  |
| Monday, July 31 | Cardinality: Part II | 13.2, 13.3 |  |
| Wednesday, August 2 | Induction: Part II | 10.1, 10.2 |  |
| Monday, August 7 | Sequences, Limits, delta-epsilon Proofs | (Supplement) |  |
| Wednesday, August 9 | Set Theory and Axiom of Choice | 1.9, 1.10, (Supplement) |  |
| Monday, August 14 | Final Review | Review Packet |  |
| Wednesday, <br> August 16 | Final Exam |  |  |

