Workshop 9

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1 Goal

The goal of this workshop is to discuss limits of functions at infinity, and its similarities with limits of sequences. We will also discuss what it means to say that the limit of a function is infinite.

2 Limits at Infinity

Definition 2.1. Let $f : \mathbb{R} \to \mathbb{R}$. We say that the limit of f at infinity is L, denoted

$$\lim_{x\to\infty} f(x) = L$$

if for every $\varepsilon > 0$, there exists $N > 0$ such that for all $x > N$, $\big|f(x) - L\big| < \varepsilon$.

1. Prove that $\lim_{x\to\infty} \frac{1}{x} = 0$.
2. Prove that $\lim_{x\to\infty} \frac{3x+1}{2x-5} = \frac{3}{2}$.
3. Prove that $\lim_{x\to\infty} \frac{3x+1}{2x^2-5} = 0$
4. Prove that $\lim_{x\to\infty} \frac{\sin(x)}{x} = 0$.
5. What about $\lim_{x\to\infty} \sin(x)$?
6. Suppose $\lim_{x\to\infty} f(X) = L$. Define a sequence $\{a_n\}$ by $a_n = f(n)$. Prove that $\lim_{n\to\infty} a_n = L$.

3 Limits of Infinity

Definition 3.1. Suppose $f : \mathbb{R} \to \mathbb{R}$ and let $a \in \mathbb{R}$. We say that the limit of f at a is infinite, denoted

$$\lim_{x\to a} f(a) = \infty$$

if for all $M > 0$, there is a $\delta > 0$ such that for all $x \in \mathbb{R}$ with $0 <|x - a| < \delta$, $f(x) \geq M$.

1. Prove that $\lim_{x\to 0} \frac{1}{|x|} = \infty$.
2. Suppose that $\lim_{x\to 0} f(x) = 0$. Prove that $\lim_{x\to 0} \frac{1}{\big|f(x)\big|} = \infty$.