Workshop 8

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1 Goal

The goal of this workshop is to practice proving statements about limits of functions.

2 Limits of Functions

1. Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be defined by $f(x) = x \sin(\frac{1}{x})$. Prove that $\lim_{x \to 0} f(x) = 0$.

2. Suppose $g : \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x) = \begin{cases} x & x \text{ is rational} \\ 0 & x \text{ is irrational} \end{cases}$$

   Prove that $\lim_{x \to 0} g(x) = 0$.

3. Let $D \subseteq \mathbb{R}$ and let x_0 be an accumulation point of D. Let $f : D \to \mathbb{R}$. Prove that if

$$\lim_{x \to x_0} f(x) = L,$$

   then for every sequence $\{a_n\}$ in D which converges to x_0,

$$\lim_{n \to \infty} f(a_n) = L.$$

4. Suppose $D \subseteq \mathbb{R}$ and let x_0 be an accumulation point of D. Consider two functins $g, f : D \to \mathbb{R}$ such that

$$\lim_{x \to x_0} g(x) = 0$$

   and f is bounded, in the sense that $\exists M > 0$ such that $|f(x)| \le M$ for all $x \in D$. Prove that

$$\lim_{x \to x_0} g(x) f(x) = 0.$$