Workshop 6

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1 Goal

The goal of this workshop is to review sequences by exploring their properties and constructing counterexamples to false statements.

2 Questions

Determine whether each of the following statements about sequences of real numbers is true or false. Then, give a proof or counterexample.

1. If $\lim_{n\to\infty}(a_n - b_n) = 0$, then $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$.
2. If $a_n \to a$, then $|a_n| \to |a|$.
3. If $a_n \to a$ and $(a_n - b_n) \to 0$, then $b_n \to a$.
4. If $a_n \to 0$, $a_n > 0$ for all n, and $|b_n - b| < a_n$ for all n, then $b_n \to b$.
5. If $a_n \to a$, where $a > 0$ and $a_n > 0$ for all n, then $\sqrt{a_n} \to \sqrt{a}$.
6. If $\{a_n\}$ converges to a, then every subsequence also converges to a.
7. If $\{a_n\}$ is a sequence such that every proper subsequence converges, then $\{a_n\}$ also converges.
8. If $\{a_n\}$ is a monotone sequence with a convergent subsequence, then $\{a_n\}$ converges.
9. Every convergent sequence is Cauchy.
10. Every bounded sequence is convergent.
11. Every convergent sequence is bounded.
12. If $\{a_n\}$ is a Cauchy sequence, then so is $\{(-1)^n a_n\}$.
13. If $\{a_n\}$ is a bounded sequence and $\{b_n\}$ is a convergent sequence, then $\{a_n b_n\}$ converges.