

# Workshop 4

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## 1 Goal

The goal of this workshop is to practice proving statements about arithmetic operations on sequences.

## 2 Questions

1. For which (if any) real numbers  $a$  and  $b$  does the sequence

$$\left\{ (a^n + b^n)^{1/n} \right\}_{n=1}^{\infty}$$

converge? For those values, what does it converge to?

2. Find an example of each of the following, or prove that one cannot exist.

- (a) Sequences  $\{x_n\}$  and  $\{y_n\}$ , neither of which converge, but such that the sum  $\{x_n + y_n\}$  converges.
- (b) A sequence  $\{x_n\}$  which converges and  $\{y_n\}$  which does not converge such that  $\{x_n + y_n\}$  converges.
- (c) A sequence  $\{b_n\}$  that converges with  $b_n \neq 0$  for all  $n$ , and such that  $\{1/b_n\}$  does not converge.
- (d) A bounded sequence  $\{a_n\}$  and a convergent sequence  $\{b_n\}$  such that  $\{a_n - b_n\}$  is bounded.
- (e) Two sequences  $\{a_n\}$  and  $\{b_n\}$  where both  $\{a_n\}$  and  $\{a_n b_n\}$  converge but  $\{b_n\}$  does not converge.

3. In this problem, we look at Cesaro means:

- (a) Prove that if a sequence  $\{s_n\}$  converges, then the sequence of averages

$$y_n = \frac{x_1 + \cdots + x_n}{n}$$

also converges to the same thing.

- (b) Give an example to show that the sequence of averages can converge even if the  $x_n$  don't converge.
4. Let  $\{a_n\}$  be a bounded (but not necessarily convergent) sequence of real numbers and let  $\{b_n\}$  be a sequence that converges to 0. Prove that  $a_n b_n$  converges to 0. Can we say anything about the sequence  $a_n b_n$  if  $b_n$  converges to some nonzero number?