Workshop 3

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1 Goal

The goal of this workshop is to practice proving some properties of sequences of real numbers.

2 Sequences

1. In this problem, we explore what happens if we reverse the order of the quantifiers in our definition of converges of a sequence.

   Definition 2.1. A sequence (x_n) **verconges** to x if there exists an $\varepsilon > 0$ such that for all $N \in \mathbb{N}$ it is true that $n \geq N$ implies $|x_n - x| < \varepsilon$.

   (a) Give an example of a vercongent sequence.
   (b) Is there an example of a vercongent sequence that is divergent?
   (c) Can a sequence verconge to two different values?
   (d) What exactly is being described in this strange denition?

2. **Squeeze Theorem:** Suppose $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$, and $\{c_n\}_{n=1}^{\infty}$ are sequences of real numbers such that for some $A \in \mathbb{R}$,

   $$\lim_{n\to\infty} a_n = A \text{ and } \lim_{n\to\infty} b_n = A.$$

   Suppose further that for all n, $a_n \leq c_n \leq b_n$. Prove that

   $$\lim_{n\to\infty} c_n = A.$$

3. Prove that if $\{s_n\}_{n=1}^{\infty}$ is a convergence sequence of real numbers, then $\{|s_n|\}_{n=1}^{\infty}$ also converges. Is the converse true? Prove or find a counterexample.

4. Suppose $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are two sequences of real numbers such that for some $A, B \in \mathbb{R}$,

   $$\lim_{n\to\infty} a_n = A \text{ and } \lim_{n\to\infty} b_n = B.$$

   Prove that

   $$\lim_{n\to\infty} (a_n b_n) = AB.$$