Workshop 1: In Class

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1 Goal

The goal of this workshop is to explore the ideas of limits, without formal definitions.

2 Playing with "Limits"

1. Suppose I define
$$x = \sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}}}}}.$$
In other words, x is $\sqrt{2}$ raised to the power $\sqrt{2}$ infinitely many times. What is x? Hint: how are x and $x^{\sqrt{2}}$ related?

2. **Zeno's Paradox:** Suppose I want to walk across a room. Zeno points out that I must first walk halfway there. But once I've reached halfway, I then have to walk half of the remaining distance, and then half of the remaining distance after that, and so on and so forth. At each point in my walk, I must first walk half of the remaining distance before I can reach the other side of the room. Therefore, I can never actually reach the other side of the room.

   But we know that it's perfectly possible for me to walk from one side of the room to the other. So what's wrong with Zeno's argument?

3. Suppose I start listing numbers $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ and so on. What happens as I continue to list numbers? Do they keep getting larger (in other words, do they go off to ∞)? Do they bunch up? If they bunch up, are they getting closer and closer to just one number? What number is that?

4. What if my list went $1, -1, 1, -1, 1, -1, \ldots$ and so on. What happens as I continue this list?

5. What about the list $\frac{1}{2}, \frac{1}{2} + \frac{1}{4}, \frac{1}{2} + \frac{1}{4} + \frac{1}{8}, \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}, \ldots$?

6. What about the list $3, 3.1, 3.14, 3.141, 3.1415, 3.14159, \ldots$? Are the numbers in this list rational or irrational? What number does this list get close to? Is that number rational or irrational?

3 Review of Cardinality

1. Suppose $A_1, A_2, \dots, A_n, \dots$ is a countable number of countable sets. Let

$$A = \bigcup_{n=1}^{\infty} A_n.$$

   Prove that A is countable. You may use the fact that a subset of a countable set is countable and that there are infinitely many prime numbers.

2. Find a bijection between the set $\{\frac{1}{n} : n \in \mathbb{N}\}$ and the set $\{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$. Hint: Recall Hilbert's infinite hotel.

3. Find a bijection between the set $(0, 1)$ and $[0, 1)$.

4. Define $A = \{X \subseteq \mathbb{N} : X \text{ is finite }\}$. Prove or disprove: A is countable.