Math 311 Section 03 | Quiz 9 | Spring 2018

Name: ____

1. (1 point) Negate the following statement: For all $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x \in \mathbb{R}, |x - x_0| < \delta$ implies $\left|f(x) - L\right| < \varepsilon$.

   ◯ For all $\varepsilon \leq 0$, there exists $\delta \leq 0$ such that for all $x \in \mathbb{R}$, $|x - x_0| < \delta$ implies $\left|f(x) - L\right| < \varepsilon$.

   √ **There exists $\varepsilon > 0$ such that for all $\delta > 0$, there exists $x \in \mathbb{R}$ with $|x - x_0| < \delta$ but $\left|f(x) - L\right| \geq \varepsilon$.**

   ◯ There exists $\varepsilon > 0$ such that for all $\delta > 0$, there exists $x \in \mathbb{R}$ with $|x - x_0| > \delta$ and $\left|f(x) - L\right| > \varepsilon$.

   ◯ There exists $\varepsilon \leq 0$ such that for all $\delta \leq 0$, there exists $x \in \mathbb{R}$ with $|x - x_0| < \delta$ but $\left|f(x) - L\right| \geq \varepsilon$.

2. (1 point) __**F**__ True or False: If $\lim_{x \to x_0} f(x) = L$ and x_0 is in the domain of f, then $f(x_0) = L$.

3. (1 point) __**T**__ True or False: If $f : \mathbb{R} \to \mathbb{R}$ is continuous at some $x_0 \in \mathbb{R}$ and $\lim_{x \to x_0} f(x) = L$, then $f(x_0) = L$.

4. (1 point) Describe the logical difference between the following two statements. (Note: simply stating that the quantifiers are rearranged is not sufficient. You should describe how that changes the logic of the statement.)

   Statement 1: For all $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x, y \in \mathbb{R}, |x - y| < \delta$ implies $\left|f(x) - f(y)\right| < \varepsilon$.

   Statement 2: For all $\varepsilon > 0$ and for all $y \in \mathbb{R}$, there exists $\delta > 0$ such that for all $x \in \mathbb{R}$, $|x - y| < \delta$ implies $\left|f(x) - f(y)\right| < \varepsilon$.

   Solution: In Statement 1, the same delta must work for all y. In Statement 2, it is possible that we require different deltas for different values of y.

5. (1 point) Determine the error in the following argument:

   Claim 1. *Every integer $n \in \mathbb{N}$ with $n > 1$, is divisible by a prime number $p < n$.*

   Proof. Let $n \in \mathbb{N}$ with $n > 1$. Let $S = \{k \in \mathbb{N} : 1 < k < n, \text{ and } k|n\}$. Since S is a subset of the natural numbers, it has a least element. Call that least element p.

   I claim p must be prime. If not, then there exists an integer q with $1 < q < p$ such that $q|p$. Since $p|n$, there is an integer a such that $n = ap$, and since $q|p$, there is an integer b such that $p = bq$. Therefore, $n = ap = abq$. Since $ab \in \mathbb{Z}$, this shows that $q|n$ and therefore $q \in S$. This contradicts our choice of p as the smallest element of S. Therefore p is prime, proving our claim. □

   Solution: The author didn't prove that S is nonempty. They are trying to use the fact that every *nonempty* subset of the natural numbers has a least element.