Math 311 Section 03

Quiz 8

Spring 2018

Name: ____

1. (1 point) Define $f : \mathbb{R} \to \mathbb{R}$ as follows:

$$f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}.$$

What is $\lim_{x\to 0} f(x)$? You do *not* need to prove your answer.

> **Solution:** $\lim_{x\to 0} f(x) = 0$

2. (1 point) __T__ True or false: Let $f : \mathbb{R} \to \mathbb{R}$ be such that f has a limit L at some $x_0 \in \mathbb{R}$. Suppose that $\{a_n\}$ is a sequence of real numbers that converges to x_0. Then the sequence $\{f(a_n)\}$ converges to L as well.

3. (2 points) For what $a \in \mathbb{R}$ does the limit $\lim_{x\to a} \lfloor x \rfloor$ exist where $\lfloor x \rfloor$ is the greatest integer less than or equal to x? You do not need to provide a formal proof, but you must give some justification for your answer. (1 point for answer, 1 point for justification)

> **Solution:** The limit exists if and only if $a \notin \mathbb{Z}$. At those points, the function jumps. (Drawing a graph is particularly helpful for this explanation)

4. (1 point) Determine the error in the following argument.

Claim 1. *Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers such that*

$$\lim_{n\to\infty} (a_n - b_n) = 0.$$

Then a_n and b_n converge to the same number.

Proof.

$$0 = \lim_{n\to\infty} (a_n - b_n) = \lim_{n\to\infty} (a_n) - \lim_{n\to\infty} (b_n).$$

Hence, by moving the last term to the left hand side, we have

$$\lim_{n\to\infty} (a_n) = \lim_{n\to\infty} (b_n)$$

□

> **Solution:** We know that the limit of the sum (or difference) is the sum (or difference) of the limits *as long as all of the limits exist.* There is no assumption here that the limits need exist, so we cannot split the limit as was done in the first line of the proof.