Math 311 Section 03 **Quiz 7** Spring 2018

Name: ____

1. (1 point) __**F**__ True or False: Every sequence of real numbers has a convergent subsequence.

2. (1 point) What is
$$\lim_{x\to 1} \frac{x-1}{x^2-1}?$$
You do *not* need to prove your answer.

Solution:
$$\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+1)} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2}$$

3. (1 point) Define $f : \mathbb{R} \to \mathbb{R}$ as follows:
$$f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}.$$
What is $\lim_{x\to 0} f(x)$? You do *not* need to prove your answer.

Solution: $\lim_{x\to 0} f(x) = 0$

4. (1 point) Suppose $f : \mathbb{R} \to \mathbb{R}$. Fill in the blanks in the definition of a limit below. Note: because the domain of f is all of $\mathbb{R}$, we do not need to worry about accumulation points.

"The limit of f at x_0 is L iff ____ such that for all $x \in \mathbb{R}$ with $0 < |x - x_0| < \delta$, $\big|f(x) - L\big| < \varepsilon$."

Solution: The limit of f at x_0 is L iff for all $\varepsilon > 0$, there exists a $\delta > 0$ such that for all $x \in \mathbb{R}$ with $0 < |x - x_0| < \delta$, $\big|f(x) - L\big| < \varepsilon$.

5. (1 point) Determine the error in the following argument:

Claim 1. *The sequence* $\{\sin(\frac{n}{2}\pi)\}_{n=1}^{\infty}$ *converges.*

Proof. Consider the subsequence $\{\sin(m\pi)\}_{m=1}^{\infty}$, namely the subsequence where $n = 2m$ is even. Since m is an integer and sine of integer multiples of pi is always 0, this sequence is identically 0, which clearly converges. $\square$

Solution: The author only proved that a subsequence converges. They have not proven that the whole sequence converges.