Math 311 Section 03 — **Quiz 6** — Spring 2018

Name: ____

1. (1 point) __**T**__ True or False: Every bounded sequence of real numbers has a convergent subsequence.

2. (1 point) __**F**__ True or False: If $\lim_{x \to x_0} f(x) = L$ and x_0 is in the domain of f, then $f(x_0) = L$.

3. (1 point) Which of the following limits **do not** exist. Select all that apply.

   √ $\lim\limits_{x \to 0} \frac{1}{x}$

   √ $\lim\limits_{x \to 0} \sin(\frac{1}{x})$

   ◯ $\lim\limits_{x \to 0} x \sin(\frac{1}{x})$

   √ $\lim\limits_{x \to 0} \sin^2(\frac{1}{x})$

   ◯ $\lim\limits_{x \to 0} x$

4. (1 point) Fill in the blanks in the definition of a convergent sequence below:

   "A sequence $\{a_n\}$ of real numbers converges to a real number A if ______________________ $\varepsilon > 0$, ______________________ $N \in \mathbb{N}$ such that $n > N$ implies $|a_n - A| < \varepsilon$."

   Solution: A sequence $\{a_n\}$ of real numbers converges to a real number A if **for all** $\varepsilon > 0$, **there exists** $N \in \mathbb{N}$ such that $n > N$ implies $|a_n - A| < \varepsilon$.

5. (1 point) Determine the error in the following argument:

   Claim 1. *The sequence* $\{\sin(\frac{n}{2}\pi)\}_{n=1}^{\infty}$ *converges.*

   Proof. Consider the subsequence $\{\sin(m\pi)\}_{m=1}^{\infty}$, namely the subsequence where $n = 2m$ is even. Since m is an integer and sine of integer multiples of pi is always 0, this sequence is identically 0, which clearly converges. □

   Solution: The author only proved that a subsequence converges. They have not proven that the whole sequence converges.