Math 311 Section 03 | **Quiz 3** | Spring 2018

Name: ______________________________

1. (1 point) Which one of the following is the correct *negation* of the definition of a sequence $\{a_n\}$ being Cauchy?

   ◯ For every $\varepsilon > 0$, there is a positive integer N such that if $m, n \geq N$, then $|a_n - a_m| < \varepsilon$.

   ◯ There exists a $\varepsilon \leq 0$ such that for all positive integer N, if $m, n < N$, then $|a_n - a_m| < \varepsilon$.

   √ **There exists a $\varepsilon > 0$ such that for all positive integers N, there exist $n, m \geq N$ such that $|a_n - a_m| \geq \varepsilon$.**

   ◯ For every $\varepsilon > 0$, there is a positive integer N such that if $m, n \geq N$, then $|a_n - a_m| \geq \varepsilon$.

2. (1 point) __**T**__ True or False: Every Cauchy sequence of real numbers converges to a real number.

3. (1 point) __**T**__ True or False: Every convergent sequence is Cauchy.

4. (1 point) Fill in the blanks in the statement of the Bolzano-Weierstrass Theorem below:

   "Every ____________________, ____________________ set of real numbers has at least one accumulation point."

   Solution: Every **bounded**, **infinite** set of real numbers has at least one accumulation point.

5. (1 point) Determine the error in the following argument:

   Claim 1. *For all $a, b \in \mathbb{N}$, if $a^2|b^2$ then $a|b$.*

   Proof. Let $a, b \in \mathbb{N}$ with $a^2|b^2$. Then, there exist an integer k such that

   $$b^2 = ka^2.$$

   Taking the square root of both sides, we get

   $$b = \sqrt{k}a,$$

   which proves that $a|b$. □

   Solution: The definition of $a|b$ requires $\sqrt{k}$ to be an integer, which the author has not proven to be the case.