Math 311 Section 03

Quiz 2

Spring 2018

Name: ____

1. (1 point) Which one of the following sequences does *not* converge?

   ○ $\{\frac{1}{n}\}_{n=1}^{\infty}$

   √ $\{\sqrt{n}\}_{n=1}^{\infty}$

   ○ $\{\frac{\sin n}{n}\}_{n=1}^{\infty}$

   ○ $\{\frac{\cos(n\pi)}{n^2}\}_{n=1}^{\infty}$

   ○ $\{\frac{2n-6}{5n+7}\}_{n=1}^{\infty}$

2. (1 point) __**T**__ True or False: Every convergent sequence of real numbers is bounded.

3. (1 point) __**F**__ True or False: Every bounded sequence of real numbers is convergent.

4. (1 point) Fill in the blank in the definition below:

   "A sequence $\{a_n\}_{n=1}^{\infty}$ of real numbers *converges* to $A \in \mathbb{R}$ iff for each $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that for all $n > N$, ______________________________ ."

   Solution: $|a_n - A| < \varepsilon$

5. (1 point) Determine the error in the following argument:

   Claim 1. *Every integer $n \in \mathbb{N}$ with $n > 1$, is divisible by a prime number $p < n$.*

   Proof. Let $n \in \mathbb{N}$ with $n > 1$. Let $S = \{k \in \mathbb{N} : 1 < k < n, \text{ and } k|n\}$. Since S is a subset of the natural numbers, it has a least element. Call that least element p.

   I claim p must be prime. If not, then there exists an integer q with $1 < q < p$ such that $q|p$. Since $p|n$, there is an integer a such that $n = ap$, and since $q|p$, there is an integer b such that $p = bq$. Therefore, $n = ap = abq$. Since $ab \in \mathbb{Z}$, this shows that $q|n$ and therefore $q \in S$. This contradicts our choice of p as the smallest element of S. Therefore p is prime, proving our claim. □

   Solution: The author didn't prove that S is nonempty. They are trying to use the fact that every *nonempty* subset of the natural numbers has a least element.