Math 311 Section 03 — **Quiz 1** — Spring 2018

Name: ____

1. (1 point) Consider the statement "$\forall a, b$, if $a < b$, then $\exists c$ such that $a < c < b$." In which of the following number systems is that statement true? Select all that apply.

   √ $\mathbb{R}$
   √ $\mathbb{Q}$
   ◯ $\mathbb{N}$
   √ $\mathbb{R} - \mathbb{Q}$
   ◯ $\mathbb{Z}$

2. (1 point) Which of the following is the negation of the statement "For each even integer n, there exists an integer k such that $n = 2k$"?

   √ **There exists an even integer n such that for all integers k, $n \neq 2k$.**
   ◯ For all odd integers, there exists an integer k such that $n \neq 2k$.
   ◯ There exists an odd integer n such that there exists an integer k with $n = 2k$.
   ◯ For all even integers n and for all integers k, $n \neq 2k$.

3. (1 point) __**T**__ True or False: Between any two distinct real numbers, there is a rational number.

4. (1 point) Fill in the blanks in the statement of the least upper bound property below:

   "Every ____________________________ set of ____________________________ numbers which is bounded above has a least upper bound."

   Solution: "Every **nonempty** set of **real** numbers which is bounded above has a least upper bound."

5. (1 point) Determine the error in the following argument:

   Claim 1. *Let $n \in \mathbb{Z}$. If $n^2 + 2n + 4$ is divisible by 4, then n is even.*

   Proof. Suppose $n \in \mathbb{Z}$ is even. Then there is an integer k such that $n = 2k$. So,

   $$n^2 + 2n + 4 = 4k^2 + 4k + 4 = 4(k^2 + k + 1).$$

   Since $k^2 + k + 1 \in \mathbb{Z}$, this shows that $n^2 + 2n + 4$ is divisible by 4. □

   Solution: This is a proof of the converse, which is *not* logically equivalent to the original statement.