# Workshop 8

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## 1 Goal

The goal of this workshop is to practice proving statements about continuity and to explore the difference between continuity and uniform continuity.

## 2 Continuity

1. Define the function $f : \mathbb{R} \to \mathbb{R}$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ in lowest terms} \\ 0 & \text{if } x \in \mathbb{R} - \mathbb{Q} \end{cases}.$$

   Prove that $f$ is continuous at every irrational point and discontinuous at every rational point.

2. Suppose the functions $f, g : \mathbb{R} \to \mathbb{R}$ are continuous and that $f(r) = g(r)$ for every $r \in \mathbb{Q}$. Prove that $f(x) = g(x)$ for every $x \in \mathbb{R}$. Hint: the rationals are dense in $\mathbb{R}$.

3. Suppose the function $g : \mathbb{R} \to \mathbb{R}$ satisfies

$$\lim_{h \to 0} (g(x + h) - g(x - h)) = 0$$

   for every $x \in \mathbb{R}$. Does that imply that $g$ is continuous? Prove or give a counterexample.

## 3 Uniform Continuity

1. Suppose $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous. Prove that for every Cauchy sequence $\{x_n\}$, the sequence $\{y_n\}$ defined by

$$y_n = f(x_n)$$

   is also Cauchy. Is this true if the function is just continuous?

2. Prove that the composition of two uniformly continuous functions is uniformly continuous. Is this true of just continuous functions?